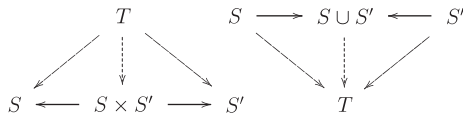


Saunders Mac Lane (1909–2005): His Mathematical Life and Philosophical Works

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Most mathematicians cannot define the spaces of algebraic geometry, called *schemes*, off the top of their heads, nor the *morphisms* mapping one scheme algebro-geometrically to another $f : S \rightarrow S'$. But they can define the cartesian product $S \times S'$ and the coproduct or union $S \cup S'$ of schemes (up to isomorphism) by familiar diagrams of morphisms:



Products and coproducts of any kind of structure are defined this way today.¹ This rigorous ‘structuralism’, where structures are defined up to isomorphism by their morphisms to and from other structures, has been textbook mathematics since Lang [1965] and Mac Lane and Birkhoff [1967]. It is in large part due to Saunders Mac Lane. He first gave these definitions of product and coproduct in Mac Lane [1948], where he emphasized that they are more fundamental than set-theoretic definitions for much work with structures like Abelian groups.

Mac Lane wrote two autobiographies. Obviously there is his *Mathematical Autobiography* [2005]. Less obvious is *Mathematics: Form and Function* [1986e] aimed at philosophers. It surveys the mathematics Mac Lane wanted philosophers to know. Naturally that is the mathematics Mac Lane wanted to know over the years; so this sweeping survey is little more sweeping than his own career. Having attended lectures by David Hilbert and Emmy Noether, and studied with Paul Bernays and Hermann Weyl, Mac Lane made noted contributions to algebra, topology, and group theory. He was influential in logic. He published on geometry and mechanics and on teaching calculus. He looked over the whole scope of mathematics as vice president of the National Academy of Sciences and

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¹ See innumerable textbooks or Mac Lane [1986e], p. 393. The left-hand diagram says there are morphisms $p_0 : S \times S' \rightarrow S$ and $p_1 : S \times S' \rightarrow S'$ such that for any scheme T and morphisms $f : T \rightarrow S$ and $g : T \rightarrow S'$ there is a unique morphism $u : T \rightarrow S \times S'$ with $p_0 u = f$ and $p_1 u = g$.

the American Philosophical Society.² He was president of the Mathematical Association of America and the American Mathematical Society, and chair of Mathematics at Chicago from 1952 to 1958 competing to be the strongest mathematics department in the world.

His central achievement was the co-creation with Samuel Eilenberg of *category theory* which became the way mathematicians organize each branch of mathematics and especially the relations between branches. *Mathematics: Form and Function* comes surprisingly close to repeating in chronological order his own mathematical interests. It develops a philosophy already recognizable in his *Journal of Symbolic Logic* reviews in the 1930s and 1940s, yet which finally relied on William Lawvere's categorical foundations (Lawvere [1963], [1964]).

Mac Lane was distinctively American. His outlook was cosmopolitan and progressive. His trademark Mac Lane tartan suit showed exuberant pride in his immigrant family past. He traced his intellectual roots to a Presbyterian grandfather 'officially charged with heresy for preaching on the writings of Charles Darwin' in Ohio in 1883 ([2005], p. 3).³ In many ways his career followed Eliakim Hastings Moore (1862–1932), who set out to make America a world force in mathematics.

Retired but still teaching, Professor Moore made a tremendous impression on beginning graduate student Mac Lane ([2005], p. 36). Both had studied at Yale. Moore studied in Germany, and headed the mathematics department at Chicago from the time the university first opened in 1892. Mac Lane went to Göttingen for his doctorate in 1931 and later headed the department at Chicago. Both were leaders in the research-oriented American Mathematical Society. Moore's interest in mathematics education made him a co-founder of the Mathematical Association of America to complement the AMS. As president of the MAA in the 1950s Mac Lane worked to update mathematics teaching. Moore was the doctoral teacher of G. D. Birkhoff, father of Garrett Birkhoff who co-authored Birkhoff and Mac Lane [1941], and Mac Lane and Birkhoff [1967]. And Moore had a principle:

The existence of analogies between central features of various theories implies the existence of a general abstract theory which

² See, e.g., the broad survey in MacLane [1980c].

³ William Ward McLane was ahead of his time. The 2002 General Assembly of the Presbyterian Church (U.S.A.) 'Reaffirms that there is no contradiction between an evolutionary theory of human origins and the doctrine of God as Creator. . . . Encourages State Boards of Education across the nation to establish standards for science education in Public Schools based on the most reliable content of scientific knowledge as determined by the scientific community.' Search 'evolutionary' on the Presbyterian Church (U. S. A.) website www.pcusa.org/ or 'Presbyterian' on the National Center for Science Education website www.ncseweb.org.

underlies the particular theories and unifies them with respect to those central features. (Moore [1908], p. 98)

Category theory was created to find the unity behind deep analogies between topology and algebra. Today it is a standard format for giving such 'general abstract' theories. The relation of Moore to Mac Lane would repay far more attention than it has gotten.⁴

Studying logic in Göttingen from 1931 to 1933, right after Gödel's incompleteness theorem, Mac Lane developed a strong feeling for Hilbert's 'formalism'. He had an exchange in the *New York Review of Books* with Freeman Dyson on the point. Dyson claimed Hilbert had gone 'into a blind alley of reductionism . . . reducing mathematics to a set of marks written on paper, and deliberately ignoring the context of ideas and applications that give meaning to the marks'. Mac Lane said to the contrary that Hilbert meant quite concrete mathematical knowledge in his famous slogan *Wir müssen wissen; wir werden wissen* (We must know, we will know): 'This was the tone of his general lecture course on the philosophy of science which I heard him give in late 1931.' Hilbert used formalization in its own proper domain:

Hilbert himself called this 'metamathematics'. He used this for a specific limited purpose, to show mathematics consistent. Without this reduction, no Gödel's theorem, no definition of computability, no Turing machine, and hence no computers.

Dyson simply does not understand reductionism and the deep purposes it can serve. Hilbert was not 'sterile'.

Dyson finally agreed with him about Hilbert and said 'Hilbert himself was, of course, a master of both [formal and concrete] kinds of mathematics', but oddly Dyson would not extend the same concession to Mac Lane. He insisted Mac Lane 'is by temperament a reductionist'. In fact Mac Lane was no kind of reductionist. He saw what formalization could achieve while he was constantly involved with the ideas and applications that give it meaning.⁵

The specific project of his Göttingen dissertation was to give formal means of abbreviating formal proofs. But the larger goal was to bring formal logic closer to practice and make it express more directly how

A proof is not just a series of individual steps, but a group of steps, brought together according to a definite plan or

⁴ Thanks to William Lawvere for emphasizing this. On Moore see Parshall and Rowe [1994].

⁵ This paragraph quotes Mac Lane [1995b].

purpose.... So we affirm that every mathematical proof has a leading idea, which determines all the individual steps, and can be given as a plan of the proof. (Mac Lane [1934], p. 60)

This larger project became a philosophical article in the *Monist* the year that Mac Lane started publishing (Mac Lane [1935]). It is reaffirmed for example in Mac Lane [1997b].

Mac Lane's most penetrating work in mathematics, and philosophy, and in history of mathematics, came from his encounter with Emmy Noether in Göttingen. He went there for Hilbert's logic and because Oystein Ore at Yale had told him of Noether who was yet little known in the U.S. He had trouble absorbing her ideas on *factor sets* and *crossed products* which replaced massive amounts of calculation in number theory with sharp conceptual arguments—although he notes 'they still involve heavy computation'.⁶ Mac Lane always said a mathematician needs to keep practiced at long calculation. Over time Mac Lane clarified factor sets and crossed products by a yet tighter conceptual framework, *group cohomology*. Noether's unification of arithmetic and abstract algebra became a unification of group theory and topology in a thoroughly categorical way. He later wrote historical expository articles with philosophic investigation of creation and conceptualization, algebra, and geometry.⁷ These ideas are omnipresent in advanced number theory today in the form of their descendants *Galois cohomology*, *étale cohomology*, and others, see, e.g., Washington [1997].

Mac Lane's most sustained pedagogical effort began with Noether as well. That was the 25-year evolution of his algebra textbook with Birkhoff. First Birkhoff and Mac Lane [1941] was an American take on the Noether school of algebra. Eventually Mac Lane and Birkhoff [1967] became many students' first look at categorical mathematics.⁸

Göttingen is also the reason, though, that philosophers today have trouble reading Mac Lane and especially his *Mathematics: Form and Function*. He pursued philosophy at length with Hermann Weyl and worked on revising Weyl [1927]. As part of his D. Phil. degree he studied philosophy of mathematics with phenomenologist and Hilbert disciple Moritz Geiger.⁹ For Göttingen mathematicians at that time philosophy was phenomenology. This phenomenology came from Edmund Husserl who had trained in mathematics with Karl Weierstrass and Leopold Kronecker and whose doctoral dissertation was on calculus of variations. Hilbert got him hired

⁶ Green *et al.* [1998], p. 870. See also Mac Lane [2005], pp. 94–98.

⁷ [1978], [1988b]. Compare his philosophical histories [1976a], [1981a], [1988a], [1989d].

⁸ It was mine. On the history of that book see Mac Lane [1997c], [1997f].

⁹ See Alexanderson and Mac Lane [1989], Mac Lane [1995a], [2005].

in philosophy at Göttingen.¹⁰ At Harvard from the mid 1930s to 1947 Mac Lane spoke often with Willard Quine but rejected Quine's 'undue concern with logic, as such' ([1986e], p. 443). Mac Lane would absorb a good dose of Karl Popper. But his base note would remain 1930s Göttingen phenomenology—the philosophy of mathematicians.

Phenomenologists accept many different attitudes towards being. Geiger especially describes the 'naturalistic attitude' which reduces everything to physical existence, and the 'immediate attitude' which recognizes social existence (as of a poem or a law), and psychic existence (as of a feeling or perception), and more. None of these is *right* or *wrong* for Geiger; rather each has its role in life. At times they compete for a role. He says some people take a naturalistic attitude towards logic, and thus reduce it to a branch of psychology, which he finds mistaken. For Geiger, mathematics can only be understood in the immediate attitude. Its objects exist as *forms* (*Gestalten*, *Gebilde*), which may be forms of physical objects, but are not themselves physically real, (Geiger [1930], esp. pp. 82, 86–88, 115).

Thus for Mac Lane mathematics studies *forms* or *aspects* which can be applied to study physically real things. All of the Euclidean and non-Euclidean geometries can be applied to physical space by suitable definitions of 'distance'. Which one best suits physics is not a mathematical question. Mathematical statements may be proved or not, illuminating or not, but they cannot be *true* or *false*, because they are not empirically falsifiable. So mathematics makes no ontological commitments. Mac Lane never became a Quinean, far less a post-Hilary Putnam Quinean.¹¹

Back in the 1930s, returned from Germany, Mac Lane focussed on algebra because it was hard to get a job in logic ([2005], p. 62). He joined the Association for Symbolic Logic and wrote a survey of topics in logic for discussion in math clubs (Mac Lane [1939a]). His logic and philosophy are scattered, often in reviews. He shows far more knowledge of proof theory in Mac Lane [1939d] than in his 1934 dissertation. He reviews many ideas on the foundations of mathematics in his [1940b] and [1942].¹² He endorses Church:

This is an up-to-date description of the central problem in the foundations of mathematics. This problem is taken to be 'the construction of a symbolic system within which the body of extant mathematics may be derived in accordance with sharply stated and immediately applicable formal rules.' Other problems, connected with the ultimate nature of mathematics or

¹⁰ See Peckhaus [1990], pp. 56f. On Weyl's phenomenology see Tieszen [2000].

¹¹ Quoting Mac Lane [1986e], p. 443; see especially pp. 91, 440–46.

¹² Authors reviewed are Ackermann, Bernays, Church, Chwistek, Gonthier, Heyting (Mac Lane [1940b]); and Barzin, Bernays, Finsler, Fréchet, Gonthier, Jørgensen, Lebesgue, Łukasiewicz, Pólya, Rohn, Sierpiński, Skolem (Mac Lane [1942]).

with the possible characterization of mathematics as a branch of formal logic, must wait on this central problem for solution. (Mac Lane [1940b])

Mac Lane too was unsatisfied with existing set theories. He would much later repeat ‘there is yet no simple and adequate way of conceptually organizing all of Mathematics’ though he considered categorical foundations the best prospect and he no longer saw this issue as prior to all others ([1986e], p. 407).

He treated themes that would reappear after he met Lawvere. Marshall Stone ‘shows surprising geometric content in many apparently purely algebraic or logical ideas’ in ways topos theory would later extend (Mac Lane [1939c], Mac Lane and Moerdijk [1992]). Work by Tarski does the same (Mac Lane [1939b]). J.-L. Destouches proposes a ‘new logic’ for physics but:

This calculus is symbolized only incompletely, and tends to confuse system with meta-system. It is not free from technical errors. . . . Any such ‘new logic’ for Physics must remain a gratuitous speculation until someone produces such a logic and uses it in Physics in some way not possible for the classical logic. (MacLane [1940a])

Later he would find (Lawvere [1979]) and related works succeed at this.

Mac Lane had great fun reviewing Eric Temple Bell’s *Development of Mathematics*. The review is worth reading today for discussion of many mathematical advances—and for Mac Lane’s defence of the philosophers: ‘Is Plato as vicious as Bell’s everywhere dense cracks would indicate? . . . The philosopher will disagree with the jabs at Kant.’ It discusses ambiguities in the idea of ‘structure’. And it conveys Mac Lane’s approach to every account of mathematics and its philosophy: ‘don’t wonder about it, but go, read, and disagree for yourself’ (Mac Lane [1946]).

From 1944 to 1948 Mac Lane was on the Council of the Association for Symbolic Logic. Around that time, as editor of the *Carus Mathematical Monographs*, he urged Stephen Kleene and Barkley Rosser to write on logic. ‘In Kleene’s case this led to the preparation of his magnificent volume *Introduction to Metamathematics*’.¹³ (Mac Lane [1996d], p. 4). He became Ph.D. advisor to logicians William Howard, Michael Morley, Anil Nerode, Robert Solovay, and Steven Awodey.

His mathematics stayed in touch with foundations. The founding paper of category theory made a startling claim.¹⁴ While giving trivial examples from linear algebra, and deep applications to extremely advanced topology,

¹³ Kleene [1952], noting Mac Lane’s encouragement and advice on drafts, p. vi.

¹⁴ Arguably the first paper on it is Eilenberg and Mac Lane [1942].

Eilenberg and Mac Lane say:

In a metamathematical sense our theory provides general concepts applicable to all branches of mathematics, and so contributes to the current trend towards uniform treatment of different mathematical disciplines. (Eilenberg and Mac Lane [1945], p. 236)

The paper also gives several strategies for the set-theoretic problems posed by large categories (Eilenberg and Mac Lane [1945], p. 246).

The great mathematical debate over category theory took place in private over the next few years, indeed in closed meetings of the Bourbaki group, with Eilenberg a member and Mac Lane sometimes visiting. He was photographed at one (Mashaal [2000], p. 85). Category theory lost in debate and won in practice. Bourbaki formulated a set-theoretic notion of ‘structure’ as basis for their encyclopedic *Elements of Mathematics* (Bourbaki [1958]) yet even their own later volumes never use this theory. Members who relied on (and invented much) category theory in their work were bitterly disappointed.¹⁵

Bourbaki structures are something like Tarski models.¹⁶ Mac Lane made at least one effort to deal with ‘structures’ in this sense since it is familiar to philosophers [1996e]. But compare [1986e], p. 33, where these are called *sets-with-structure* and treated as important examples but not the only kind of ‘structure’. He could never really engage with versions of ‘structuralism’ which neglect the tools mathematicians use. To say the least, mathematicians handle structures by the *morphisms* between them which are neglected by philosophical ‘structuralists’. To say the truth, mathematicians handle morphisms by category theory.

His encounter with Lawvere brought Mac Lane back to a focus on logic, foundations, and philosophy of mathematics.¹⁷ When he first heard of Lawvere’s ideas on set theory, which became the *Elementary theory of the category of sets* or ETCS for short, he found them entirely implausible. He found the axioms entirely persuasive when he learned them. Unfortunately, in one of those misplaced jokes that so complicate the history of thought,

¹⁵ The debate is described in detail using Bourbaki’s internal newsletter in Corry [1996], pp. 376ff. See also Mashaal [2000], pp. 52–54.

¹⁶ The difference is in morphisms. The natural morphisms of models are elementary embeddings. Bourbaki structures are higher order, and morphisms preserve higher types and may preserve fixed parameters (such as real numbers).

¹⁷ Lawvere points out some errors regarding himself in Mac Lane [2005]: He went to Indiana 13 years after Eilenberg had left and he picked up the basics of category theory there on his own; Truesdell did not suggest Columbia but helped him get in (p. 191). Lawvere did not lose his Columbia fellowship but went to California to study logic; his dissertation axiomatized the category of categories and not of sets (p. 192). He cotaught mechanics with Mac Lane because he had previously got the idea for synthetic differential geometry from Grothendieck’s work (p. 244).

Mac Lane was fond of calling ETCS ‘set theory without elements’ ([2005], at length on p. 192). Set theory is no joking matter. A good number of logicians have taken him literally, notwithstanding that elements are defined early in any account of ETCS (Lawvere [1964], p. 1507). Mac Lane has consistently backed ETCS as the best working foundation for mathematics (Mac Lane [1998a], Appendix).

To philosophers, though, he says there is no single ‘foundation’ for mathematics and the purported foundations are better seen as ‘proposals for the organization of Mathematics’ ([1986e], p. 406). They cannot be ‘starting points’ or ‘justifications’ for mathematics which is far older and more securely known than they are.

Mac Lane had used category theory to simplify many aspects of mathematics. Lawvere brought a radical idea that even the simplest notions—such as the natural numbers, or sets of functions—have simple categorical versions which could bring foundations closer to normal practice. Foundations could use the very tools that mathematicians use daily. Mac Lane jumped onto this program. He regarded it as marking a deep shift in the direction of category theory and part of a larger shift away from generality and towards focussed themes in abstract algebra. See especially his [1981a] and [1988a].

This brings us up to Mac Lane’s first autobiography, written ‘as a background for the philosophy of Mathematics’ ([1986e], back cover).¹⁸ Like his career it begins with the foundations of algebra plus the basic topology Mac Lane learned from Hausdorff’s *Mengenlehre*: ‘the first serious mathematical text that I read’ (Alexanderson and Mac Lane [1989], p. 6). It moves to axiomatic geometry, group theory, and Galois theory, all close to themes of his 1930’s research; then calculus and linear algebra which he taught in the 1940s; and then complex analysis and mechanics which he began to teach in the 1950s and 1960s. (I find no trace of his war work in applied mathematics.) It ends with foundations and philosophy. Of course that puts philosophy out of order. He studied it early and late. The book is not meant to be chronological. It is just such a powerfully, personally informed, view of mathematics that it naturally follows his course. The later autobiography [2005] has great personal detail and great mathematics. Indeed it has more advanced mathematics than *Mathematics: Form and Function*.¹⁹ But *Mathematics: Form and Function* is what Saunders had to say to the philosophers.

Let him have the last word, from a talk he gave at mid-career, fifty-one years ago:

It cannot be too often reiterated that the aim of collegiate mathematics is the understanding of mathematical ideas per se. The applications support the understanding, and not vice versa . . .

¹⁸ An early sketch makes somewhat different points (Mac Lane [1981b]).

¹⁹ And a fine personal, historical, and conceptual preface by David Eisenbud.

We must contrive ever anew to expose our students—be they general students or specialized students—to the beauty and excitement and relevance of mathematical ideas. We must set forth the extraordinary way in which mathematics, springing from the soil of basic human experience with numbers and data and space and motion, builds up a far-flung architectural structure composed of theorems which reveal insights into the reasons behind appearances and of concepts which relate totally disparate concrete ideas. . .

Mathematics as it is today . . . can no longer be presented by piecemeal courses, for it is simply no longer true that advanced mathematics can be split neatly into compartments labelled ‘algebra,’ ‘analysis,’ ‘geometry,’ and ‘applied mathematics’ . . .

You may protest that I am talking about research and not about education. . . I answer, first: The character and direction of current research is the best indication of the ideas which we ought to be teaching. I answer, second: One of our main responsibilities is that of training the research mathematicians of the future. American Mathematics has made tremendous strides [by] the infusion of European mathematical talent. In the decades to come, we must produce a similar infusion on our own and from our own students. . .

The proper treatment of calculus for functions of several variables requires vector ideas; the budding statistician and the coming physicist need them; modern analysis is unthinkable without the notion of linear dependence and all that flows from it. Throughout these courses the infusion of a geometrical point of view is of paramount importance. A vector is geometrical; it is an element of a vector space, defined by suitable axioms—whether the scalars be real numbers or elements of a general field. A vector is not an n -tuple of numbers until a coordinate system has been chosen. Any teacher and any text book which starts with the idea that vectors are n -tuples is committing a crime for which the proper punishment is ridicule. The n -tuple idea is not ‘easier,’ it is harder; it is not clearer, it is more misleading. By the same token, linear transformations are basic and matrices are their representations. . . . (Mac Lane [1954], pp. 152–154)

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three 60 minute videotapes for the American Mathematical Society, Providence, Rhode Island: ‘Algebra as a means of understanding mathematics’ (a joint AMS-MAA lecture, Columbus, Ohio, August 1990), ‘Fifty years of Mathematical Reviews’ (a joint AMS-MAA lecture, Louisville, Kentucky, January 1990), ‘Some major research departments of mathematics’ (a joint AMS-MAA invited address, Providence, Rhode Island, August 1988).

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