Category Theory for Computing Science: Update

Michael Barr and Charles Wells

February 28, 1994

We intend to maintain our text, **Category Theory for Computing Science**, Prentice Hall, 1990 (ISBN 0-13-120486-6), in the sense that programmers maintain their programs, by keeping a list of known errors and also of additions to the text that we think might be useful. (The latter will probably come in spurts as we go to meetings and find out about wonderful new applications of category theory to computing science.)

We will periodically announce new errata and addenda on the category theory bulletin board. The latest TeX version of the complete list is available by e-mail or p-mail from either author.

The update consists of three parts: 1: A list of errors discovered so far. 2: Additional examples, problems and pointers to the literature. 3: Additions and updates to the list of references, pp. 417ff. of the text. Page references refer to the text.

Any further corrections or suggestions for additional text will be welcome.

Michael Barr
Department of Mathematics
Burnside Hall
McGill University
805 Sherbrooke St. West
Montréal, P. Q., Canada H3A 2K6
barr@triples.math.mcgill.ca

Charles Wells
Department of Mathematics
Case Western Reserve University
University Circle
Cleveland, OH 44106-7058, USA
cfw2@po.cwru.edu

1 Errors

Anders Gammelgaard has pointed out that we label identity arrows in diagrams in three different ways. The identity arrow on an object A can be labeled id_A, A, or $=$. We have not attempted to insert corrections making the usage uniform, but we thought it worthwhile to call the readers' attention to the variation, which does in fact illustrate variations found in the literature.

p. xv In the Chapter Dependency Chart, there should be a diagonal line from Chapter 5 to Chapter 7.

p. 12 (Todd Turnidge). Line 5: $f \in \phi_1(a)$ should read $f \in \phi_0(a)$.

p. 23 (Jean-Pierre Marquis). SM–2 should read, "If m, $n \in S$, then $mn \in S$".

p. 30 (Rick Rarick). Lines 1 and 2: The sentence beginning at the end of line 1 should say "Then f^* is the underlying function of a homomorphism of monoids…".

p. 31 (Jean-Pierre Marquis). S–1 should say $\mathcal{D}_0 \subseteq \mathcal{C}_0$ and $\mathcal{D}_1 \subseteq \mathcal{C}_1$.

p. 32. Line −2: The reference to 3.1.6 should be a reference to both 2.8.10 and 3.1.6.

p. 34 (Nils Andersen). Line 16: "$f \neq f'$ or $g \neq g'$" should be "$f \neq g$ or $f' \neq g'$

p. 34 (Nils Andersen). Third line above 2.6.10: "a **indexed**" should be "an **indexed**".

p. 43 (Nils Andersen). Lines 3 through 5 are nonsense. Replace them by the statement: "The relation $\sim$ is symmetric by definition."

p. 47 (Nils Andersen). Line −5: "The left inverse" should be "A split monomorphism".

p. 52 (Nils Andersen and Anders Gammelgaard). The second sentence of 3.1.2 should read, "On objects, a homomorphism f must take the single object of $C(M)$ to the single object of $C(N)$, and F–1 is trivially verified since all arrows in $C(M)$ have the same domain and codomain and similarly for $C(N)$."

p. 54 (Anders Gammelgaard). Line 4 of 3.1.13: Change "so we are forced" to "so it is reasonable".

p. 54 (Andrew Malton). Line 8: read "$g \circ h = f$" for "$h \circ g = f$"

p. 56 (Han Yan). Line 7: $g : C \longrightarrow C$ should be $g : B \longrightarrow C$.

p. 57 (Han Yan). Line 1: $f : A \longrightarrow A$ should be $f : C \longrightarrow A$.

p. 57 (Han Yan). Line 2: $g : B \longrightarrow C$ should be $g : B \longrightarrow D$.

p. 61 (Rick Rarick). The paragraph after FA–2 should read: Recall that the free monoid $F(A)$ is the set A^* with concatenation as multiplication. The function ϕ^* as just defined is thus an action of $F(A)$ on S (Exercise 2).

p. 63 (Jean-Pierre Marquis). Last line: $TF(S)$ should be $FT(S)$.

p. 64 (Jean-Pierre Marquis). Definition 3.3.2, line 4: $G(f)$ should be $F(g)$.

p. 66 (Jean-Pierre Marquis). Section 3.3.10, line 2: "is to said" should be "is said".

p. 73 (Jean-Pierre Marquis). The reference to Wells [1989] should be deleted.

p. 68 CF–1 should read: On objects, $C(h)(M) = N$.

p. 74 (Jean-Pierre Marquis). The reference to Wells [1989] should be to Wells [1990] (see updated reference in Section 3 of this document).

p. 79 (Al Vilcius). Line 7: replace "id_i" by id_{D_i}.

p. 79 Line 3: "diagram" should be capitalized.

p. 83 (Andrew Malton). Line −7: change "of shape U" to "of shape $\mathcal{U}$".

p. 87 (Al Vilcius and Anders Gammelgaard). In Theorem 4.2.20, line 2, replace "$\mathcal{C}$" by "$\mathcal{G}$ in $\mathcal{D}$". In the last line replace "$\mathbf{Mod}(\mathcal{G},\mathcal{C})$" by "$\mathbf{Mod}(\mathcal{G},\mathcal{D})$".

p. 90 (Andrew Malton). Line 5: change "$f : \mathcal{E} \longrightarrow V\mathcal{G}_1$" to "$f : \mathcal{E} \longrightarrow \mathcal{G}_1$"

p. 92. In Proposition 4.3.12 and its proof, the letter C (not the script $\mathcal{C}$) is used with two different meanings. This can be corrected by changing C to S in the first line of the proposition, third line (first occurrence only), fourth line (last occurrence only) and in the first line of the proof (second occurrence only).

p. 96. The reference to Seely should be [1987] (the entry in the list of references, p. 425, was incomplete and is updated in the list of references below.)

p. 101 (Jean-Pierre Marquis, Han Yan). In the figure, $\mathrm{Hom}(f, C)$ should be $\mathrm{Hom}(C, f)$.

p. 102. "The" not "the" in line 7.

p. 103 (Al Vilcius). Change "set" to "collection" in the second line of 4.6.2 and the sixth line of 4.6.3. [We are using "collection" as an informal synonym for "class", without getting into set theory.]

p. 103. Between the fourth and fifth lines of 4.6.3, add the following: "We write $M : \mathcal{S} \longrightarrow \mathcal{C}$ for such a model. This use of the same symbol to denote both the sketch homomoprhism and the graph homomorphism is a bit of notational overloading that in practice is always disambiguated by context."

p. 104 (Nils Andersen). The second sentence of Example 4.6.6 should be deleted.

p. 105 (Nils Andersen). Line 4: "with an arrow $s : N \longrightarrow A$" should be "with arrows $s : N \longrightarrow A$ and $\mathrm{id}_N : N \longrightarrow N$"

p. 105 (Anthony Bucci). To the diagrams in (4.24) should be added the diagram

p. 105 (Nils Andersen). The first sentence of the second paragraph should read: "The surprise is that a homomorphism $\alpha : M \longrightarrow M'$ of models of this sketch must take the particular loop $M(s)(n)$ to $M'(s)(\alpha N(n))$."

p. 105. The sentence before 4.6.9 has a misplaced parenthesis. It should read "If you want a sketch for graphs which have a loop on every node, but not a distinguished loop (so that a homomorphism takes the loop on n to some loop on $\alpha N(n)$, but it does not matter which one), you will have to wait until we can study regular sketches in 9.4.5."

p. 105 (Jean-Pierre Marquis). LT–1, second line: a should be f.

p. 107 (Todd Turnidge) Change "is a homomorphism of models" to "is (the only component of) a homomorphism of models".

p. 108 (Al Vilcius). Change "set" to "collection" in line 3 of 4.7.2.

p. 108 (Nils Andersen). In the second line of Definition 4.7.3, "$\mathcal{C}$" should be "**Set**".

p. 109 (Nils Andersen). The heading for 4.7.6 should be "Term models".

p. 110 (Anders Gammelgaard). Two lines above I.1: $\mathcal{C}$ should be $\mathcal{G}$ and "described" should be "defined".

p. 124 (Anders Gammelgaard). Line 9 of the proof of Lemma 5.2.15: $\mathrm{Hom}_{\mathcal{C}}(V, A \times B)$ should be $\mathrm{Hom}_{\mathcal{C}}(-, A \times B)$

p. 124 (Todd Turnidge) The left arrow of Diagram (5.10) is defined as in 5.2.13 and the right arrow is defined as in 3.1.21.

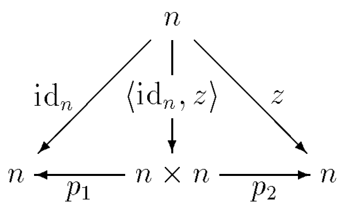
p. 124 (Anders Gammelgaard). Line 11 of the proof of Lemma 5.2.15: $p_{A,B}(q)(p_1, p_2)$ should be $P_{A,B}(q)(p_1, p_2)$.

p. 131 (Nils Andersen). Lines 8 and 9 of 5.3.12 should be *replaced* by the following: "the property that $P.A \circ \langle f, g\rangle = f$ and $P.B \circ \langle f, g\rangle = g$. This would make P".

p. 137 (Jean-Pierre Marquis). Line 3 of proof of Theorem 5.5.5: $u_i \circ s$ should be $s \circ u_i$.

p. 165 (Nils Andersen). In Diagram (a), the left arrow from 1 to n should go from n to 1.

pp. 165–166 (Anders Gammelgaard). The recursive equations given here do *not* define the natural numbers. It is an interesting exercise to work out just what they do define. Here is a correction. In Example 7.1.7, a number of changes must be made to reverse $\langle \mathrm{id}_n, z\rangle$ to $\langle z, \mathrm{id}_n\rangle$. The third line under "We will also need new arrows:" should be $\langle \mathrm{id}_n, z\rangle : n \longrightarrow n \times n$. Diagram (b) should be changed to:



$\langle \mathrm{id}_n, z\rangle$ should be changed to $\langle z, \mathrm{id}_n\rangle$ in Diagram (e). The first set-off formula on page 166 should be

$$M\langle \mathrm{id}_n, z\rangle = \langle M(\mathrm{id}_n), M(z)\rangle = \langle \mathrm{id}_{M(n)}, M(z)\rangle$$

Finally, the line just above the beginning of 7.1.8 should say $k + \mathrm{zero} = k$ instead of $\mathrm{zero} + k = k$.

p. 166 (Nils Andersen). In Diagram (f), $\mathrm{id} \times \mathrm{succ}$ should be $\mathrm{id}_n \times \mathrm{succ}$.

p. 169 (Jodelle Wuertzer). In diagram (d), the $s \times s$ on the left should be s and the s on the right should be $s \times s$.

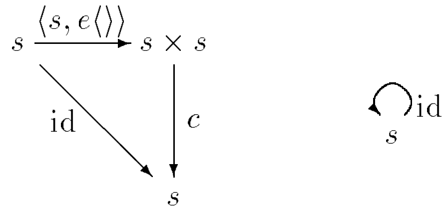
p. 170 (Anders Gammelgaard). Two lines below (vii): "limit cones" should be "product cones".

p. 173 (Jean-Pierre Marquis). The second line of N–5 should be

$$f_1 \times f_2 \times \cdots \times f_n : a_1 \times a_2 \times \ldots \times a_n \longrightarrow b_1 \times b_2 \times \cdots \times b_n$$

(no angle brackets around the arrow).

p. 174–175 (Anders Gammelgaard). The shape graph at the bottom of the page and the top of the next are nonesense, since the definition of commutative diagram requires that any two paths from one node to another in the diagram be equal. The way to deal with this situation is with two diagrams for each identity. For the first identity, we have



and the second is similar.

p. 177 (Anders Gammelgaard). Last line: "limit cone" should be "product cone".

p. 180 (Anders Gammelgaard). Line 9 of section 7.6.5: "cones C of $\mathcal{C}$" should be "cones C of $\mathcal{L}$".

p. 180 (Anders Gammelgaard). Last line: $[C\langle\rangle]$ should be $[C()]$.

p. 181 (Anders Gammelgaard). Line 1: FP-4 should begin "If for…".

p. 183 (Anders Gammelgaard). Line −8: "collection M_f of functions" should be "collection of functions M_f".

p. 184 (Anders Gammelgaard). In Example 7.7.3, the variable t occurring in (iii) and in the following two lines should be changed to z.

p. 184 (Jean-Pierre Marquis). In the next to last line of Example 7.7.3, "sort τ" should be "sort σ".

p. 186 (Jean-Pierre Marquis and Anders Gammelgaard). Lines 3, 4, 5 and 6 of 7.7.7 should be replaced by the following: "signature S and the operations f and m be as in 7.7.3. Let t be the term $f(y, m(x, y), z)$." In line 8, replace $xyxt$ by $xyxz$. In line 10, replace $\{x, y, t\}$ by $\{x, y, z\}$. Let u be the term x of arity 'σ' and sort σ and variable list 'x'. We will work out the diagram corresponding to the equation $t = u$, that is $f(y, m(x, y), z) = x$. The path $\mathbf{Q}(t)$ is". The string "'yxyt'" at the end of the line following the display should be replaced by "'yxyz'".

p. 186 (Jean-Pierre Marquis). In line 8 of Example 7.7.7, the path of u is $\mathrm{id} : \sigma \longrightarrow \sigma$.

p. 190 (Anders Gammelgaard). In FE–2 and FE–4, replace "*)" by "$*$)".

p. 190 (Nico Verwer).There is a series of errors in FE–5 to FE–8. The correct versions are as follows:

FE–5. $+\circ\langle \mathrm{id}, 0\circ\langle\rangle\rangle = +\circ\langle 0\circ\langle\rangle, \mathrm{id}\rangle = \mathrm{id} : f \longrightarrow f$

FE–6. $*\circ\langle j, j\circ 1\circ\langle\rangle\rangle = *\circ\langle j\circ 1\circ\langle\rangle, j\rangle = j : u \longrightarrow f$

FE–7. $+\circ\langle \mathrm{id}, -\rangle = +\circ\langle -, \mathrm{id}\rangle = 0\circ\langle\rangle : f \longrightarrow f$

FE–8. $*\circ(j\times j)\circ\langle \mathrm{id}, ()^{-1}\rangle = *\circ(j\times j)\circ\langle ()^{-1}, \mathrm{id}\rangle = 1\circ\langle\rangle : u \longrightarrow u$

p. 198 (Anders Gammelgaard). Lines 12 and 18: $\{\}$ should be $\{\emptyset\}$.

p. 201 (Nils Andersen). Line 3 of Definition 8.1.2. In addition to the property mentioned here, an equalizer $j : A \longrightarrow A$ of $f, g : A \longrightarrow B$ must also have the property that j equalizes f and g. This will follow from the rest of the definition if there is *some* arrow that equalizes f and g, but there needn't be such a thing.

p. 205 (Nils Andersen). The fourth sentence of 8.2.6 should read: "An object of this category is a commutative cone $\{p_i : C \longrightarrow Di\}$ and an arrow to $\{p'_i : C' \longrightarrow Di\}$ is an arrow $f : C \longrightarrow C'$ such that $p'_i \circ f = p_i$ for all i." The sixth sentence should read: "A terminal object in cone(D), if one exists, is a commutative cone over D to which every other commutative cone over D has a unique arrow."

p. 208 (Al Vilcius). Top diagram should have a B, not C in the SE corner. The fourth line should say that p_2, not p_1 is the pullback of f along g.

p. 208 (Nils Andersen). Fourth line under top diagram: "that p_1 is" should be "that p_2 is".

p. 209 (Anders Gammelgaard). Line 12 should begin "This is a family..."

p. 211 (Nils Andersen). Line −6: p_1 should be p_2.

p. 221 (Anders Gammelgaard). The example at the bottom half of the page is wrong. We seem to have stumbled upon an instance in the category of monoids of a sum that is universal. Not all are, though. Replace **N** by the monoid (group, in fact) **Z** whose elements are all the powers, positive, negative and zero, of a single generator x. Then $\mathbf{Z} + \mathbf{Z}$ has two generators x and y and all words in their positive, negative and zero powers. For the vertical arrow, take the homomorphism $\mathbf{Z} + \mathbf{Z} \longrightarrow \mathbf{Z}$ that takes each generator to the generator of the image. The words that go to the identity consist of all the words in the powers of x and y for which the sum of the exponents is 0. It includes, for example, any word of the form $x^n y^{-n}$. On the other hand, the arrow restricted to the summands takes only the identity element to the identity.

p. 222 (Anders Gammelgaard). At the end of the third line of 8.6.9, replace "$B \longrightarrow B \times_A A_i$" by "$\sum B \times_A A_i \longrightarrow B$".

p. 224 (Anders Gammelgaard). Four lines above Theorem 8.7.4: $m(b, t(u, v))$ should be $m(b, t(u, w))$.

p. 224ff (Paul Taylor). Theorem 8.7.4 is incorrect as stated. It should conclude merely that there is a coequalizer *in the category of free algebras*. The error comes from the reference on l. 2 of 226 to exercise 3, which says that a map between free algebras for a free theory is either epimorphic or factors through a proper subset of any free generating set. The problem is solved and from the solution, although not from the statement, it is clear that this is true only in the category of free algebras and Taylor has given a counter-example for the category of algebras. The statement has to be reworded, but so does the proof because it has to be reworded as a construction of a coequalizer in the Kleisli category. The details are quite easy and are left to the reader.

p. 224ff. We claim that this proof can be modified to use transfinite induction for the infinite case. This is not clear since a colimit (even one along a countable chain) of free algebras is not generally free. Lacking a convincing argument, the statement should be modified to apply to finitely generated free algebras for a free theory generated by finitary operations.

p. 225. Change V to Z in Diagram 8.13.

p. 226 (Anders Gammelgaard). The line just under (8.17): Exercise 2 does not apply here. Here is an exercise that does (the proof is easy).

If d^0, $d^1 : A_1 + A_2 \longrightarrow B$ is a parallel pair, with $u_i : A_i \longrightarrow A_1 + A_2$ the coproduct injections for $i = 1, 2$, if $e : B \longrightarrow C$ is a coequalizer of $d^0 \circ u_1$ and $d^1 \circ u_1$ and if $c : C \longrightarrow D$ is a coequalizer of $e \circ d^0 \circ u_2$ and $e \circ d^1 \circ u_2$, then $c \circ e : B \longrightarrow D$ is a coequalizer of d^0 and d^1.

p. 226 (Anders Gammelgaard). The parallel pair of arrows in diagram 8.16 should be $d^0 \circ g$ and $d^1 \circ g$, respectively.

p. 227 (Jean-Pierre Marquis). Line 2 of Exercise 2 should have B_i for B and C_i for C.

p. 228 (Al Vilcius). Last line: Word "be" is missing.

p. 229 (Nils Andersen). In the diagram in N–8, p should be $a \times_c b$.

p. 232 (Jean-Pierre Marquis). The first line of the second paragraph of section 9.1.5 has a font error (it is not mathematically wrong): the second parenthesized expression should read $(D, \mathrm{mult}_D, \mathrm{unit}_D)$

p. 234. In the binary trees example at the bottom of the page, we also need a node 1, in addition to the ones listed on line −8.

p. 235 (Nils Andersen). In the first figure, d should be n.

p. 235 (Nils Andersen). The left diagram near the bottom of the page, the one containing the arrow labeled "false", is redundant and should be omitted. The phrase "be diagrams" directly underneath that diagram should become "be a diagram".

p. 236 (Frank Piessens) The statement, "It would be incorrect to assume in general (although it is true in familiar examples) that every node of $\mathcal{T}$ is a limit of a diagram in the graph of $\mathcal{S}$" is wrong. The reason is that $\mathcal{T}^{\mathrm{op}}$ is a full subcategory of the category $\mathbf{Fun}(\mathcal{S}, \mathbf{Set})$ (to be precise, that should be the free category generated by the graph underlying $\mathcal{S}$) and every functor is a colimit of representables and, hence, in $\mathcal{T}$, every functor is a limit of them. Thus every object of the theory is a limit of a diagram built from the objects and arrows of $\mathcal{S}$.

p. 243 (Anders Gammelgaard). In lines 9 and 10, replace $\langle\rangle$ by $()$, referring to the empty path as defined on page 13.

p. 244 (Nils Andersen). In line 6 of 10.2.3, the sentence "We use nodes t^+, t and d" should read "We use nodes $1, t^+$, t and d".

p. 244 (Anders Gammelgaard). The description of the binary trees sketch is incomplete. Add that we need the cones and cocones necessary to say that $t^+ = 1+t$ with inclusions empty and incl and that $t^+ = t \times d \times t$ with projections left, val and right.

p. 244 (Nils Andersen). In the diagram at the bottom of the page, **Stack** should be **BinTree** and G should be H.

p. 246 (Anders Gammelgaard). In the first line of 10.2.6, change "category" to "sketch".

p. 248, 249 (Anders Gammelgaard). In the statements of both 10.3.2 and 10.3.3, change $\mathcal{T} \longrightarrow \mathcal{S}$ to $\mathcal{S} \longrightarrow \mathcal{T}$.

p. 249 (Nils Andersen). Line 7 should read "For any sketch $\mathcal{S}$, let $\mathbf{Mod}_{\mathcal{C}}(\mathcal{S})$ be the category of models of $\mathcal{S}$ in $\mathcal{C}$ (we called this $\mathbf{Mod}(\mathcal{S}, \mathcal{C})$ in Section 9.1)."

p. 254 (Anders Gammelgaard). On line −10, change $u : C'' \longrightarrow C'$ to $u : C'' \longrightarrow C$.

p. 255 (Al Vilcius). Line −5: Replace "cleavage" by "opcleavage".

p. 256 (Al Vilcius). Line −8: the expression has misplaced brackets. Should be

$$Fg[Ff(u)] \circ \kappa(g, Ff(X)) \circ \kappa(f, X)$$

p. 256 (Al Vilcius) Line -2 should be "functors $\mathcal{C}^{\text{op}} \longrightarrow \mathbf{Cat}$."

p. 258 (Al Vilcius). Definition 11.2.3: GS–1 should be "object of $G_0(\mathcal{C}, F)$".

p. 259 (Al Vilcius). Line 8 should read "$x' = Ff(x)$".

In paragraph starting line 11, orientation of $G_0(\mathcal{C}, F)$ requires the functor $G_0(F)$ to be the projection on the second coordinate, not first.

p. 259 (Anders Gammelgaard). The reference to Exercise 4 at the end of a line in the middle of the page should be to Exercise 2.

p. 261 (Al Vilcius). In Theorem 11.2.9, first projection should be second as above.

p. 262 Line 2 of 11.2.10: For notational consistency, the split fibration generated by $\mathcal{C}$ and G should be denoted $\mathsf{F}(\mathcal{C}, G)$ rather than $\mathcal{F}(\mathcal{C}, G)$.

p. 262 (Anders Gammelgaard). 11.2.10: The rules FC–1 through FC–3 correctly describe $\mathsf{F}(\mathcal{C}, G)$ (see previous correction), but the sentence "When the Grothendieck construction is applied to a functor $G : \mathcal{C}^{\text{op}} \longrightarrow \mathbf{Cat}$, the result is a split fibration $\mathsf{F}(\mathcal{C}, G)$" is incorrect. It is more precisely $\mathsf{F}(\mathcal{C}^{\text{op}}, G^{\text{co}})^{\text{op}}$, where $G^{\text{co}}(C)$ is defined to be $G(C)^{\text{op}}$, and, for $f : C \longrightarrow D$ in $\mathcal{C}$, $G^{\text{co}}(f) = G(f)^{\text{op}} : G(D)^{\text{op}} \longrightarrow G(C)^{\text{op}}$.

p. 263 (Jean-Pierre Marquis and Han Yan). Line 15: Change "then $F(C)$ and $G(C)$" to "then $F(C)$ and $F(D)$".

p. 264 (Anders Gammelgaard). Line 3: $\zeta(\kappa(f, x)) = \kappa'(f, x)$ should be $\zeta(\kappa(f, X)) = \kappa'(f, \zeta(X))$.

p. 264 (Anders Gammelgaard). Last line: $P'u = \text{id}_C$ should be $P'(\zeta u) = \text{id}_C$.

p. 264 (Al Vilcius). In Definition 11.3.4, FI–1, replace "$P : \mathcal{E} \longrightarrow \mathbf{Cat}$" by "$P : \mathcal{E} \longrightarrow \mathcal{C}$".

p. 265 (Anders Gammelgaard). Line 2: Change both instances of f to id_D.

p. 265 (Anders Gammelgaard). Line 5: Change $\hat{u} \circ \kappa'(f, X) = \kappa(f, X') \circ \zeta u$ to $\hat{u} \circ \kappa'(f, \zeta(X)) = \kappa(f, \zeta(X')) \circ \zeta u$.

p. 265 (Anders Gammelgaard). Line 7: Change $\tilde{u}$ to $\zeta \tilde{u}$,

p. 265 (Anders Gammelgaard). (11.9) should read

$$\begin{aligned}\hat{u} \circ \kappa'(f, \zeta(X)) &= \kappa'(f, \zeta(X')) \circ \zeta(u) \\ &= \zeta(\kappa(f, X')) \circ \zeta(u) \\ &= \zeta(\kappa(f, X') \circ u) \\ &= \zeta(\tilde{u} \circ \kappa(f, X)) \\ &= \zeta(\tilde{u}) \circ \zeta(\kappa(f, X)) \\ &= \zeta(\tilde{u}) \circ \kappa'(f, \zeta(X))\end{aligned}$$

p. 265 (Anders Gammelgaard). In the second line of GR–2, $(\alpha F x, C)$ should be $(\alpha C x, C)$.

p. 266 (Anders Gammelgaard). The last line: $S(G, \mathcal{B})(f) : G(A') \longrightarrow G(A)$ should be $S(G, \mathcal{B})(f) : \mathbf{Fun}(G(A'), \mathcal{B}) \longrightarrow \mathbf{Fun}(G(A), \mathcal{B})$.

p. 267 (Anders Gammelgaard). First line: The following clause should be added to the sentence ending on this line: "and it takes a natural transformation $\alpha : H \longrightarrow H' : G(A') \longrightarrow \mathcal{B}$ to the natural transformation $\alpha G f : H \circ G f \longrightarrow H' \circ G f : G(A) \longrightarrow \mathcal{B}$ whose component at an object X of A is the component of α at $Gf(X)$." This is the construction given in 4.4.2, page 94.

p. 270 Line 3: (Anders Gammelgaard). "techniques" is misspelled.

p. 271 (Nils Andersen). Line −3: "monoid to be" should be "monoid M to be".

p. 272 (Anders Gammelgaard). In the third line of the proof of 12.1.2, change the equation to $u = Ug \circ \eta X$.

p. 271 Line −3: "We define a monoid to be" should be "We define a monoid M to be".

p. 272 Line 3: (Anders Gammelgaard). The first four lines of this page should be replaced by: "A homomorphism from a monoid M to a monoid N is a function $f : UM \longrightarrow UN$ which preservers the operation and the identity element. The underlying functor is defined on homomorphisms by $Uf = f$." The notation (M, f, N) we introduced for homomorphisms was never used.

p. 272 (Jean-Pierre Marquis). Line −4: The reference to 12.1.2 should be to 12.1.1.

p. 273 (Anders Gammelgaard). On the line in the middle of the page just below the diagram and also two lines below, replace Uh by h and id_{X^*} by id_{FX}.

p. 279 (Nils Andersen). In the diagram, the diagonal arrow should be labeled $\langle q_1, q_2 \rangle$.

p. 281 (Jean-Pierre Marquis). In Exercise 1, $f(S_0) \subseteq T_0$ should be $f_*(S_0) \subseteq T_0$.

p. 282 (Anders Gammelgaard). Replace $\mathrm{id} \circ g$ by $g \circ \mathrm{id}$ (a nice example of the difference intensional and extensional equality).

p. 282 (Anders Gammelgaard). The last sentence of 12.3.3 should read, "It follows from the Yoneda embedding, Theorem 4.5.3, that $FA \cong F'A$." Then add, "The naturality of the latter isomorphism follows from the next theorem."

p. 285, 286 (Anders Gammelgaard). 12.4.1 is badly organized. First the functor Σ_f should be defined and then f^* is defined (in the presence of pullbacks) on objects and its mapping property verified; nd then 12.3.4 puts it all together into a functor.

p. 286 (Anders Gammelgaard). In line 2, change "from p_1' to p_1" to "from p_1 to p_1'".

p. 287 (Jean-Pierre Marquis). The second line should read

$$\mathrm{Hom}_{\mathcal{C}/A}(u \times v, w) = \mathrm{Hom}_{\mathcal{C}/A}(\Sigma_u(u^*(v)), w)$$

p. 291 (Anders Gammelgaard). The 0 at the left end of (13.1) should be $\emptyset$. Replace the paragraph at the bottom of the page by, "The crucial point is that when R is finitary, it commutes with the colimit that defines Z so that RZ is the colimit of the sequence

$$R(\emptyset) \longrightarrow R^2(\emptyset) \longrightarrow R^3(\emptyset) \cdots$$

which is canonically isomorpic to Z. The structure map $z : RZ \longrightarrow Z$ is this isomorphism. The proof that this works is fairly technical and is omitted."

p. 295 (Anders Gammelgaard). The definition of polynomial functor, 13.2.8, is not entirely clear. Here is a better formulation. "The class of polynomial endofunctors on a category with finite sums and products is the least class that contains the constant endofunctors and the identity functor and is closed under the operations of finite products and sums."

p. 302. Line -4ff should say, " In particular, **Cat** is enriched over **Cat** itself, since its hom sets are themselves categories (with the arrows as objects and the natural transformations as arrows) and the hom functors preserve the extra structure."

p. 303 (Jean-Pierre Marquis). In the second line of SP–4, the E should be A.

p. 307 (Jean-Pierre Marquis). In line 5 of the second paragraph, $(a_0, a_1, a_2, a_4 \ldots)$ should be $(a_0, a_1, a_2, a_3 \ldots)$.

p. 310 (Nils Andersen). In the first diagram, $\mathrm{Sub}(k' \circ k)$ should be $\mathrm{Sub}(k \circ k')$.

p. 311 (Anders Gammelgaard). The reference to 4.5.12 on line 3 should be to 4.5.8.

p. 311 (Nils Andersen). In Example 14.1.3, χ should be defined by

$$\chi(x) = \begin{cases} \text{true} & \text{if } x \in S_0 \\ \text{false} & \text{if } x \notin S_0 \end{cases}$$

p. 315 (Anders Gammelgaard). On line 8, replace "from A_0 to A'_0" by "from A'_0 to A_0.

p. 317 (Anders Gammelgaard). Exchange "true" and "false" in the diagram or else in the third and fourth lines following it.

p. 319 (Jean-Pierre Marquis). G_0 and G in the diagram should be $\mathcal{G}_0$ and $\mathcal{G}$.

p. 322 (David Benson) In 14.5.7, we should have defined a presheaf E on a poset P to be nearly constant if whenever $0 < x \leq y$ in P, the restriction $E(y) \longrightarrow E(x)$ is an isomorphism.

p. 322 In line 12 of 14.5.7, $E(x_i) \longrightarrow E(x_i) \wedge E(x_j)$ should be $E(x_i) \longrightarrow E(x_i \wedge x_j)$.

p. 322 The third sentence of 14.5.7 should read: "It turns out that every nearly constant presheaf over P is a sheaf over P if and only if the meet of two nonzero elements of P is nonzero."

p. 322 (Anders Gammelgaard). The phrase "elements of P" on line -11 is clearly wrong. Just "elements" would be better, but the reader might want to know, "elements of what?" One answer might be some kind of universal set, but even if that made sense it isn't quite right because two elements that were distinct at one level might become indistinguishable at a lower level. This is only motivation and perhaps should not be taken too seriously.

p. 331 (Jean-Pierre Marquis). Line 2: "That of (C)" should be "That of (c)".

p. 333 (Jean-Pierre Marquis). In line 4 of the paragraph after REAL–2, $[\![fa_1 = fa_2]\!]$ should be $[\![\phi a_1 = \phi a_2]\!]$.

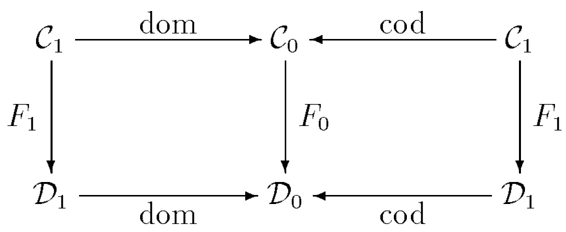
p. 334 (Nils Andersen). Line 11: $Q(A)$ should be $Q(a)$.

p. 335. In the fourth paragraph of the discussion of the internal category of modest sets, the sentence, "We want to describe this subset as consisting of those relations that are symmetric and transitive", should have the following phrase added: "and double negation closed".

p. 345. Line 2 of the answer to exercise 12.a: the last letter should be "B", not "b".

p. 357 (Jean-Pierre Marquis) In the top diagram, one of the arrows from BOOLEAN to BOOLEAN should be reversed.

p. 357 (Stephen J. Bevan) The second diagram from the bottom should be



p. 358 (Jean-Pierre Marquis) In the answer to Exercise 5, delete the phrase "β satisfies".

p. 365 In the answer to Exercise 8, $\beta C : \mathrm{Hom}(C, C) \longrightarrow F(C)$.

p. 366 In the answer to Exercise 10, the last line should say that $g \circ f = \mathrm{id}_C$ and $f \circ g = \mathrm{id}_{C'}$.

p. 368 (Jean-Pierre Marquis) In the second line of the answer to Exercise 3, the B's should be C's.

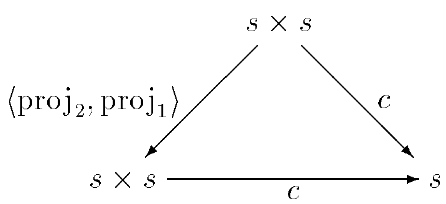
p. 370 (Jean-Pierre Marquis) Line -2: Both occurrences of f should be F.

p. 372 (Jean-Pierre Marquis) In the answer to Exercise 1, the arguments to f are reversed. The end of the sentence should read: "…by letting $f(b, 0) = f_0(b)$ and having defined $f(b, i)$ for $i \leq n$, define $f(b, n + 1) = t(f(b, n))$".

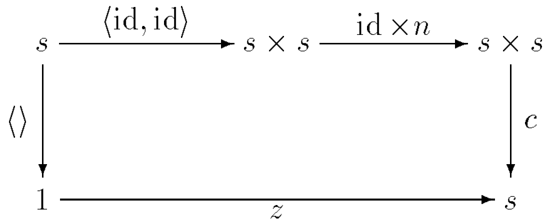
p. 373 (Jean-Pierre Marquis) In the answer to Exercise 1 of Section 6.1, every occurrence of $(\lambda \circ \mathrm{eval})$ should be $\lambda(\mathrm{eval})$.

p. 380 (Jean-Pierre Marquis) In the diagram in the answer to Exercise 2, the upper right arrow (labeled $\langle \mathrm{id}, e \circ \langle \rangle \rangle$) should go in the opposite direction.

p. 381 (Jean-Pierre Marquis) The answer to number 3 is confused. Here is the entire answer rewritten: We give the answer for the case of real vector spaces; the other is similar. We assume the real number field $\mathbf{R}$ as a given structure. We suppose, for each $r \in R$, a unary operation we will denote $r^* : s \longrightarrow s$. We require a unit element $z : 1 \longrightarrow s$ for the operation c and a diagram similar to the previous exercise to say that z is the unit element. The following diagram says that c is commutative:



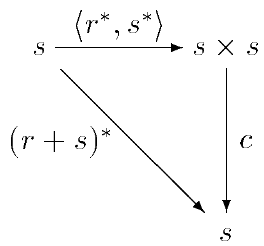
We have to add a unary negation operator $n : s \longrightarrow s$ together with a diagram to say it is the negation operator:



In addition, we need diagrams that express the following identities:

$$\begin{array}{rcl} 0^*(x) & = & z \\ r^*(c(x,y)) & = & c(r^*(x), r^*(y)) \\ (r+s)^*(x) & = & c(r^*(x), s^*(x)) \\ 1^*(x) & = & x \\ r^*(s^*(x)) & = & (rs)^*(x) \end{array}$$

We give, for example, the diagram required to express the third of the equations above:



p. 386 (Jean-Pierre Marquis) Line 4 of answer to 3b: D_m should be Dm.

p. 387 (Nico Verwer). The solution to Exercise 4 of 8.3 contains several errors. The right hand label of the diagram should be $\langle f, g\rangle$, the lower left corner of the diagram should be B and the $f \times g$ in the displayed line that begins $G(X) = \ldots$ should also be $\langle f, g\rangle$.

p. 388 (Nils Andersen). After the change is made on p.208, the answer to number 6 should read: "In **Set**, the pullback is

$$P = \{(a,b) \mid f(a) = g(b)\}$$

Since f is surjective, for any $b \in B$, there is a $a \in A$ such that $f(a) = g(b)$. Then $p_2(a,b) = b$. Thus p_2 is surjective."

p. 402 (Jean-Pierre Marquis). The answer to Exercise 6 of 12.2 (page 281) uses Theorem 12.3.6, which comes in a later section. Here is an answer using only the definition of adjoint: If the functor defined in 12.2.4 has a left adjoint F, then by definition of left adjoint there is for any set X an arrow $\eta X : X \longrightarrow FX \times A$ with the property that for any function $f : X \longrightarrow Y \times A$ there is a unique function $g : FX \longrightarrow Y$ for which $(g \times A) \circ \eta X = f$. Now take $Y = 1$, the terminal object (any one element set). There is only one function $g : FX \longrightarrow 1$, so there can be only one function $f : X \longrightarrow 1 \times A \cong A$. If A has more than one element, this is a contradiction.

p. 425 (Anders Gammelgaard). On line 4, replace "Nielson" by "Nielsen".

p. 431 The word "relation" should also be indexed on p. 21.

p. 432 (Anders Gammelgaard). The word "monad" should be indexed on p. 297.

2 Additional text

p. xiii (First paragraph). Several new books on category theory have appeared since our text was published. New books specifically concerning applications to computing science include [Asperti-Longo, 1991] (particularly strong on connections with logic), [Gunter, 1992], and [Pierce, 1991]. New texts on aspects of category theory include [Freyd-Scedrov, 1990], [Mac Lane-Moerdijk, 1992] and [McLarty, 1992]

(Additional comment). The reader may find the following discussions of the uses of category theory in computing science useful: The tutorials in [Pitt, Abramsky, Poigné and Rydeheard, 1986], and [Goguen, 1991].

p. xiii (Second paragraph). Other collections of papers are [Pitt, Rydeheard, Dybjer, Pitts and Poigné, 1989] and [Fourman, Johnstone and Pitts, 1992].

p. 17 [Additional example of category]. Let α be a relation from a set A to a set B and β a relation from B to C (see 1.3.4). The **composite** $\beta \circ \alpha$ is the relation from A to C defined as follows: If $x \in A$ and $z \in C$, $(x, z) \in \beta \circ \alpha$ if and only if there is an element $y \in B$ for which $(x, y) \in \alpha$ and $(y, z) \in \beta$. With this definition of composition, the **category of sets and relations** has sets as objects and relations as arrows.

p. 53 Add to third paragraph of 3.1.7: Freyd-Scedrov [1990], pages 9 and 19, give a formal definition of forgetful functor.

p. 71 [New exercise]. Show that the category of sets and relations is equivalent, in fact isomorphic, to its own dual (see 2.6.7).

Answer: Let **Rel** denote the category of sets and relations. For a relation α from A to B, that is, a subset of $A \times B$, let $\alpha^{\mathrm{op}} \subseteq B \times A$ be the subset $\{(b, a) \mid (a, b) \in \alpha\}$. Let $F : \mathbf{Rel} \longrightarrow \mathbf{Rel}^{\mathrm{op}}$ be the identity on objects and for a relation $\alpha : A \longrightarrow B$, let $F(\alpha) = \alpha^{\mathrm{op}}$. $F(\alpha)$ is a relation from B to A in **Rel**, hence a relation from $F(A) = A$ to $F(B) = B$ in $\mathbf{Rel}^{\mathrm{op}}$. It is easy to check that if $\beta : B \longrightarrow C$, then $F(\beta \circ \alpha) = \alpha^{\mathrm{op}} \circ \beta^{\mathrm{op}}$ in **Rel**, so $(\beta \circ \alpha)^{\mathrm{op}} = \beta^{\mathrm{op}} \circ \alpha^{\mathrm{op}}$ in $\mathbf{Rel}^{\mathrm{op}}$. This says $F(\beta \circ \alpha) = F(\beta) \circ F(\alpha)$, so F preserves composition. The identity relation on A is $\Delta_A = \{(a, a) \mid a \in A\}$, so $\Delta = \Delta^{\mathrm{op}}$ and F preserves identities. Since for any relation α, $(\alpha^{\mathrm{op}})^{\mathrm{op}} = \alpha$, we have $F \circ F$ is the identity functor on **Rel**, so is its own inverse. Hence F is an isomorphism. By Exercise 1, it is therefore an equivalence of categories.

p. 96 The applications of 2-categories have mushroomed and include [Ji-Feng and Hoare, 1990], [Moggi, 1989] and [Power, 1989] in addition to the papers already listed.

p. 97 Second paragraph of Section 4.5: In addition to generalizing the Cayley Theorem, the Yoneda Lemma also has as a special case the embedding of a poset into its down-closed subsets. Also: set-valued functors are studied further in Sections 11.2 and 14.4.

p. 142 The text by Gunter [1992] gives a systematic treatment of programming language semantics in terms of the ideas of this chapter.

p. 158 The assumption that every object in a cartesian closed category has fixed points is inconsistent with other desirable assumptions on the category. See [Huwig-Poigné, 1990].

p. 213 Freyd-Scedrov [1990] have a different but closely related definition of "regular". If a category is regular in our sense it is regular in theirs, and if it is regular in their sense and has coequalizers, then it is regular in our sense.

p. 214 Add the following exercise:
Show that $h : B \longrightarrow C$ is a coequalizer of $f,\ g : A \longrightarrow B$ if and only if for each object D,

$$\mathrm{Hom}(C, D) \xrightarrow{\mathrm{Hom}(h, D)} \mathrm{Hom}(B, D) \underset{\mathrm{Hom}(g, D)}{\overset{\mathrm{Hom}(f, D)}{\rightrightarrows}} \mathrm{Hom}(A, D)$$

is an equalizer.

p. 218 Add the following exercise:
Show that the cocone $\{u_a : Da \longrightarrow C \mid a \in \mathrm{ob}(\mathcal{G})\}$ is a colimit cocone if and only if for each object B, the cone $\{\mathrm{Hom}(u_a, B) : \mathrm{Hom}(C, B) \longrightarrow \mathrm{Hom}(Da, B)\}$ is a limit cone in sets. This is often abbreviated

$$\mathrm{Hom}(\operatorname{colim} Da, B) \cong \lim \mathrm{Hom}(Da, B)$$

p. 237 Further work on generalizations of sketches are in [Barr-Wells, 1992] and [Power-Wells, 1992].

p. 255 Rosebrugh and Wood [1992] apply indexed categories to relational databases and Cockett and Spencer [1992] use them in studying datatypes.

p. 257 Besides [Coquand, 1988], Moggi [1989] also uses the Grothendieck construction in modeling polymorphism.

p. 283 Another reference for the Adjoint Functor Theorem (with a different point of view) is [Freyd-Scedrov, 1990], pages 144-146.

p. 287 Just before the exercise: See also [Ehrhard, 1989].

p. 301 Some applications of triples (monads) in computing science are in [Moggi, 1991], [Power, 1990], [Cockett-Spencer, 1992], and [Wadler, 1992].

p. 309 General references on toposes include [Barr-Wells, 1985], [Johnstone, 1977], [Mac Lane-Moerdijk, 1992] and [McLarty, 1992].

p. 318 Add comment: We considered presheaves as actions in Section 3.2. They occur in other guises in the categorical and computer science literature, too. For example, a functor $F : A \longrightarrow \mathbf{Set}$, where A is a set treated as a discrete category, is a "bag" of elements of A. If $a \in A$, the set $F(a)$ denotes the multiplicity to which a occurs in A. See [Taylor, 1989] for an application in computing science.

p. 320 Another reference for sheaves and their connection with logic is [Mac Lane–Moerdijk, 1992].

p. 325 Goguen [1990] has developed a sheaf semantics for object oriented programs.

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