Control Insect Pests Problem

from exercise 1.4, Britton N.F.

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Understanding the problem

Outline One way to control insect population is to use $s$terile insect technique, which is a method of using genetics to control birth rate of insects. When the male insects are sterile, sterile males have to compete with the wild-type male to mate the females. If a female mates with a sterile male then they will have no offspring. The outcome of this control is the reduction of the next generation's population.

Prob 1.4 (a)

Equation: $N_n + 1 = f(N_n) = R_0 N_n(\frac{N_n}{N_n+S})(\frac{1}{1+aN_n})$
Where:
N_n denotes the number of population in the n-th generation.
R_0 denotes the basic reproductive ratio, R_0 >1.
S denotes the population of sterile insects.
a is a constant, a > 0.
Note that without the term $\frac{N_n}{N_n+S}$, the equation is just the Hassel equation of exact compensation. However, the survival fraction not only depends on the competition for limited resource but also depends on the probability of mating with non-sterile population.
Therefore, the survival fraction is $(\frac{N_n}{N_n+S})(\frac{1}{1+aN_n})$
Also, the term $\frac{N_n}{N_n+S}$ represents the probability of a given insect picks a wild-type mate, i.e., fertile fraction.

Prob 1.4(b)

Solving for $N^* = f(N^*)$:
$N^* = R_0 N^* (\frac{N^*}{N^*+S})(\frac{1}{1+aN^*})$

$N^* + S = \frac{R_0 N^*}{1+aN^*}$

If we solve for S in term of N^*, we would have:

$S(N^*) = N^*(\frac{R_0-1-aN^*}{1+aN^*})$ Eq.(I)

If we solve for $N*$ in term of S, we would have:

$N^*(S) = -(\frac{1+Sa-R_0 \pm \sqrt{(1-2Sa-2R_0+S^2a^2-2SaR_0+R_0{}^2)}}{2a})$ Eq.(II)

Also considering the plot of f(N,S), we know that for low positive S, the graph of f(N,S) has two equilibrium points. However, as S goes above a particular value, the graph of f(N,S) has only one trivial (0,0) equilibrium. We illustrate bifurcation in Figure 1 below.

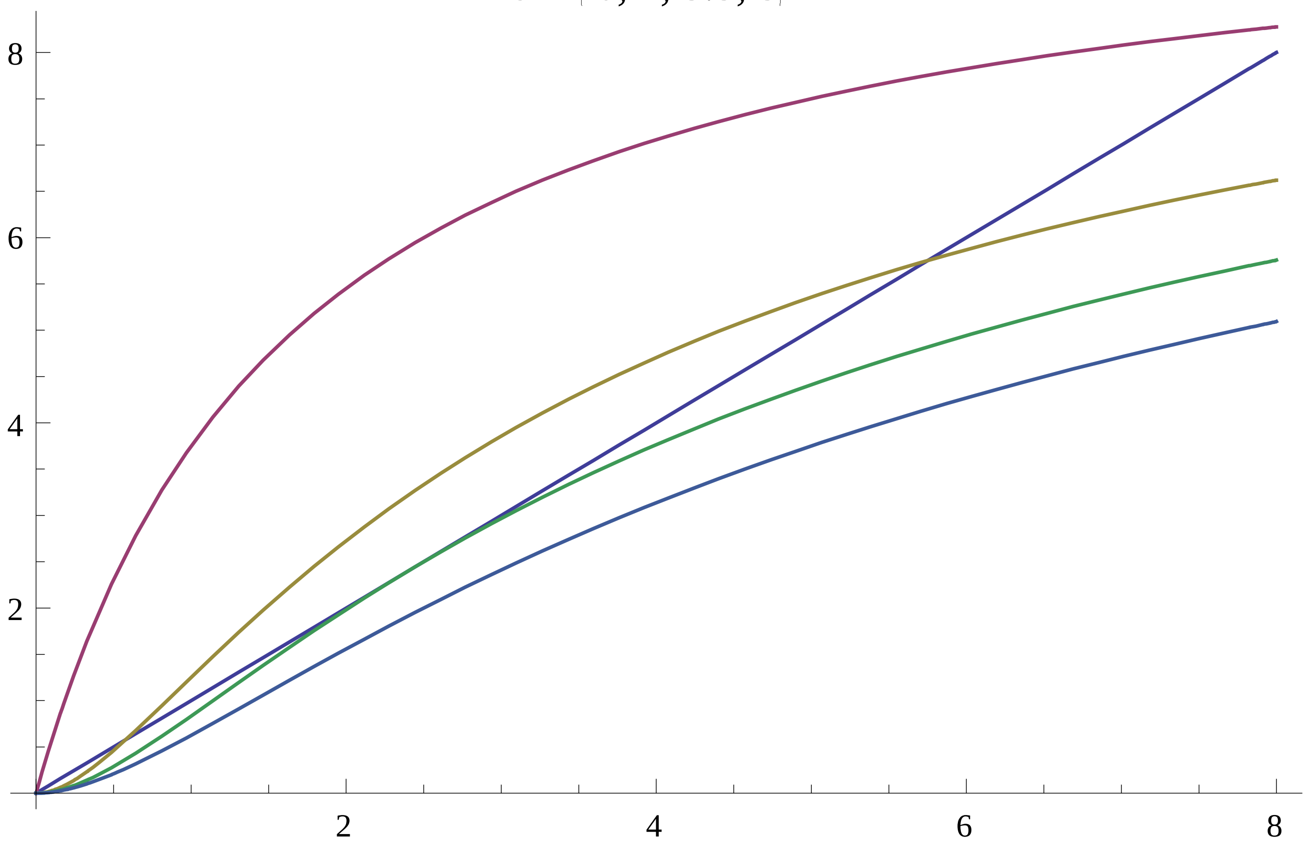
Prob 1.4(c)

Here, we have two ways to solve for the least value of S which can drive the insect population goes to extinction. When it comes to extinction, the only equilibrium point of the system is the trivial point at $N^* = 0$.

- **Solving for critical point of Eq.(II):** though Eq.(II) represents two roots, ${N_1}^*(S)$ and ${N_2}^*(S)$, of the equation $f(N^*) = N^*$, by differentiating both with respect to S, we obtain the same critial point S and that S is a bifurcation point.
- **Solvign for critical point of Eq.(I):** we can solve more easily this way since we only have one equation to deal with. Figure.2 shows the graph of S where we set $R_0 = 8 anda = 0.3$. We want the value of S such that the system has only one trivial steady state. From Figure.2, we know that the point of maximum S.
  $\frac{dS}{dN^*} = 0$
  $N^* = \frac{-1 \pm sqrtr}{a}$

R0 = 6

s : {0, 2, 3.5, 5}



One value of N^* is negative and does not make sense. Therefore, we use $N^* = \frac{-1+sqrtr}{a}$ substituting to Eq.(I) and obtain maximum S, which is equal to 11.14 for a =0.3 and $R_0 = 8$. Therefore, $S_c = 11.14$, the least S that drives the population to extinction state.

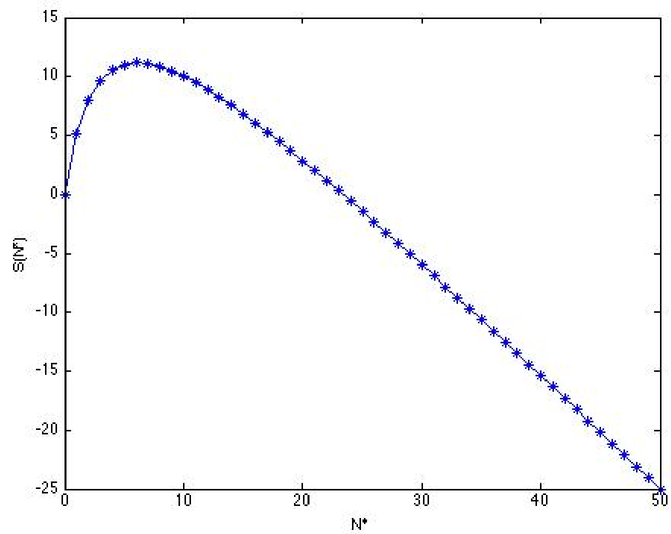


Figure 2: plot of S(N^*)

1 Prob 1.4(d)

. The cobweb plots display below. They show when $S > S_c$ the only steady state is the trivial $N^* = 0$.

2 cases : $S < S^*$, and $S > S^*$