M445: Review and term paper

David Gurarie

Spring 2001

1 PDE models in physics, geometry et al

1. Balance-conservation laws (traffic flow)
2. Fluxes and balance (Heat-diffusion problems)
3. Variational calculus and Hamilton's "Least action" principle (minimal length and surface area, vibrating strings, membranes)
4. Other examples and models: gas-fluid dynamics, acoustics, Maxwell's equations of E-M fields

2 First order pde's: method of characteristics

1. The types and meaning of boundary conditions
2. Well posed problems: uniqueness, stability
3. Linear 1-st order pde's and characteristics
4. Quasilinear pde's and characteristics; solution breakup. Shocks.
5. Application to 1D waves, and traffic flow

3 Second order pde's in 1D

1. 1D waves; characteristics and d'Alambert solution (traveling waves); sources and forces
2. Energy conservation
3. 1D diffusion: point sources and Gaussian
4. 1D problems with dissipation and transport
5. The reflection method for the heat and wave problems of half-line and interval

4 Boundary value problems in 1D; Sturm-Liouville problem and eigenfunction expansion

1. Separation and S-L problem
2. Types and meaning of boundary conditions (radiation/absorption)
3. Examples of S-L problems
   (a) constant coefficient differential operator L; Fourier/trig. series
   (b) Legendre polynomials
   (c) Hermite functions
4. Self-adjointness, completeness and orthogonality, convergence of eigenfunction expansion
5. Plancherel formula and energy conservation
6. Series solutions of the heat and wave problems, continuous and boundary sources
7. Stationary and periodic equilibria; resonance
8. Operator formalism (formal solutions)
9. Green's functions of 1D problems on line, half-line, interval
10. Eigenfunction expansion vs. reflection method!

5 Multi-D boundary value and eigenvalue problems

1. Harmonic functions, examples (holomorphic functions et al); Mean-value, Maximal principle, uniqueness
2. Series solutions and Poisson kernel in the rectangle, box, disk, solid ball
3. Eigenvalue problem for the Laplacian in the box (multiple Fourier series)
4. Eigenvalue problem in the disk
5. Spherical harmonics and Legendre DE
6. Eigenvalue problem in the 3D ball
7. Completeness, orthogonality, expansion, series solutions of multi-D heat and wave problems

6 Green's functions

1. Green's identities
2. Laplacian in $\mathbb{R}^n$, potentials
3. Green's function and Poisson kernel in regions with boundaries
4. Reflection method for the half-space, disk, ball
5. Heat-diffusion and Gaussian in $\mathbb{R}^n$ and half-space
6. Wave propagators in 3D and 2D (method of descent). Hueguens principle.

7 Term paper (due May 1)

There are 6 basic topics above covered in the class. Your term paper should discuss 3 of them: any one of 1-2 (your choice); topic 3, topic 4 ("Boundary value in 1D...") and any one of 5-6 (your choice). The discussion should address all points raised in the review topic (list them by numbers). Specifically

- the general setup: physical models, etc.
- statement of problems, models; basic results and principles
- solution method and techniques, underlying mathematical concepts and ideas
- 1-2 typical examples to illustrate your discussion. You have to set up examples, give solution, explain method, but don't need to bring all technical (computational) details!
- Make comments, comparison, discuss pro's and con's of alternative approaches, etc.

Use good English and be concise in your writing. I expect 2-4 pp per topic to be a reasonable size. If necessary you could supplement your paper by plots, diagrams, tables, hand-written or made on *Mathematica* and/or other tools.