

Summary: Eigen modes $\begin{cases} \Delta \Psi + \lambda \Psi = 0 \\ \text{B.C.} = 0 \end{cases}$

Disk



$$\Psi_{km}(r, \theta) = R_{km}(r) e^{\pm i m \theta}$$

Ball



$$\Psi_{klm}(r, \phi, \theta) = R_{lk}(r) Y_m^l(\phi, \theta)$$

Eigenvalue $\lambda = z^2$ determined by Radial Bessel

$$R_{km} = J_m \left(z_{mk} \frac{r}{a} \right)$$

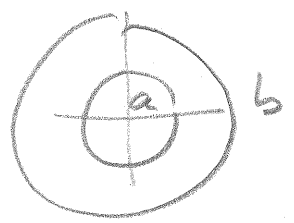
$$R_{lk} = J_{l+1/2} \left(z_{lk} \frac{r}{a} \right)$$

Eigenvalue parameter = "Bessel root"

$$z_{mk} = \begin{cases} J_m(z) = 0 & (D) \\ J'_m(z) = 0 & (N) \\ \dots & (M) \end{cases}$$

$$z_{lk} = \begin{cases} J_{l+1/2}(z) = 0 & (D) \\ J'_{l+1/2}(z) = 0 & (N) \end{cases}$$

Annulus
 $a < r < b$



$$R = A J_m(zr) + B Y_m(zr)$$

Shell

$$a < r < b$$



$$D) \Rightarrow \begin{bmatrix} J_m(za) & Y_m(za) \\ J_m(zb) & Y_m(zb) \end{bmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow$$

Charact eq-n for z

$$\det = J_m(za) Y_m(zb) - J_m(zb) Y_m(za) = 0$$