Logistic growth with harvest

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General form (Lab. 3.1): $\frac{dp}{dt} = k\left(1 - y / N\right) y - b$; (I) harvest b -const; (II) $b\left(1 + \sin \omega t\right)$ - periodic

1)Rescale variables and parameters: p→y=p/N; t→t=t/k; b→ $b \to h = \frac{b}{kN}$. Derive rescaled equation

$$\frac{dy}{dt} = \left(1 - y\right) y - h$$

2) Study bifurcation problem for const h>0; find bifurcation value h*=? sketch bifurcation diagram and solution curves for h<h*; h=h*; h> h*. Describe what happens to population y(t) is 3 cases.

3) Study over-harvested case: h>h*, and find time t_1 it takes to wipe out the population starting with initial value y_0 = 1 (hint: show $t_1 = \int_0^1 \frac{dy}{h - \left(1 - y\right) y}$, use Mathematica to compute it). Show that the total yield H =h*t $_1$(h). Find which strategy gives higher H low h over longer time t_1 or higher h over shorter t_1. Plot t_1 and H as functions of h.

4) Study numerically (NDSove, DEPlot) periodic harvest: $b\left(1 + \sin \omega t\right)$. Fix ω and let b vary from 0 to a critical value b* (?) (note it may be different from h* of (2)!). Describe the resulting solution patterns {y(t)}, and what kind of 'bifurcation phenomena' you observe in the periodic case. Estimate numerically b*. Study what, if any effect has ω on b*.