M224

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I. Parameter identification

II. Model validation / prediction

US population

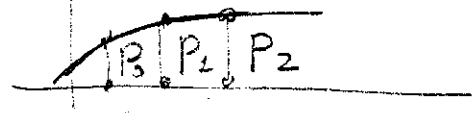
Ye	Pop
1790	P_0
⋮	⋮

Initial value

1) Linear: $P' = \alpha P$? $\longrightarrow P(t) = P_0 e^{\alpha t}$?

To find α take any pair of data points:

$$P_1 = P_0 e^{\alpha \Delta t} \longrightarrow \boxed{\alpha = \frac{1}{\Delta t} \ln \frac{P_1}{P_0}}$$



2) Logistic: $P' = \alpha P(1 - P/N)$? $\rightarrow P(t) = \dfrac{NP_0}{P_0 + (N-P_0)e^{-\alpha t}}$

Two parameters: α, N – ?

Take 2 eq-ns for 3 data p-ts: $\{P_0\ P_1\ P_2\}$

$$(*)\begin{cases} P_1 = \dfrac{NP_0}{P_0 + (N-P_0)e^{-\alpha \Delta t}} \\ P_2 = \dfrac{NP_1}{P_1 + (N-P_1)e^{-\alpha \Delta t}} \end{cases} \longrightarrow \text{?} \longrightarrow \text{Solve for } (N, \alpha)$$

or $\left(\frac{1}{N}; x = e^{-\alpha \Delta t}\right)$

Problem: (a) Show solution of (*): $x = \dfrac{1/P_1 - 1/P_2}{1/P_0 - 1/P_1}$; $N = \dfrac{1-x}{1/P_1 - x/P_0}$; $\alpha = \ldots$

(b) Derive logistic eq-ns (α, N) for US popul. data using first 3 data p-ts (1790 ...), and the do it for 3 p-ts in the middle (1850, ...)

(c) Compare 2 solutions & their predictions of the current pop.