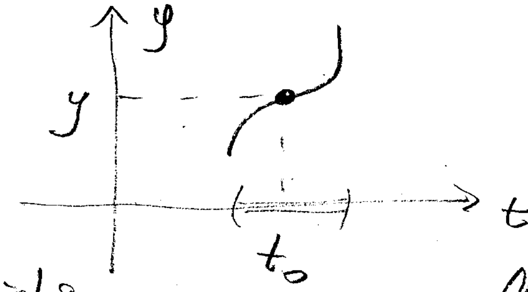
224

D. Gurarie

Uniqueness / Existence

Take IVP $\begin{cases} y' = f(y,t) \\ y(t_0) = y_0 \end{cases}$



ask whether solution exists, unique, all t?

Regular point (t_0, y_0) means both f-ns $f(y,t)$ & $\partial_y f(y,t)$ — continuous (finite) at (around) $(t_0\, y_0)$.

Thm: If $(t_0\, y_0)$ — regular, then local solution exists a neighborhood of (t_0, y_0)

Examples:

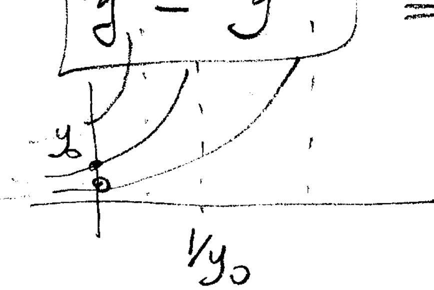
1) Linear $\boxed{y' = ay + b} \Rightarrow \boxed{y(t) = -\frac{b}{a} + \left(y_0 + \frac{b}{a}\right)e^{at}}$

all $(t_0\, y_0)$ - regular, all $-\infty < t < \infty$

---

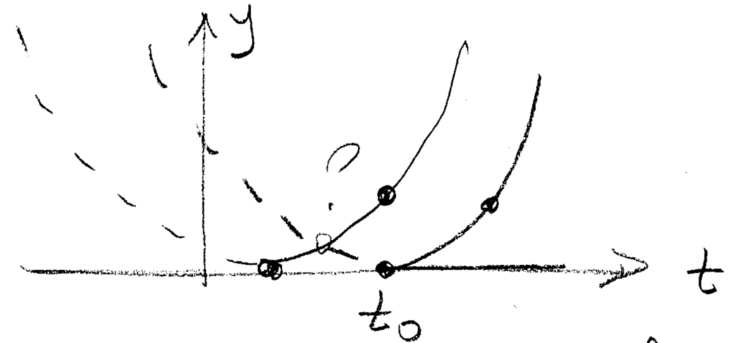
2) Finite-time "blow-up": $\boxed{y' = y^2}$ ← NL

separation $\Rightarrow \ldots \boxed{y(t) = \dfrac{y_0}{1 - y_0 t}}$



3) Nonunique solution (?) $\boxed{y' = \sqrt{y}}$

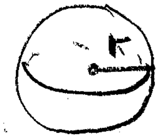
$\dots \Longrightarrow \boxed{y = (t-t_0)^2} \overset{GS}{\underset{?}{\Longleftarrow}} \quad y(t_0) = 0 \overset{?}{\Longrightarrow} \boxed{y=0}$



$\Longrightarrow$ Line $\boxed{y=0}$ - singular

$f_y(0) = \infty$!

Physical model: droplet (condensation evaporation):



r - radius

$A = 4\pi r^2$ - area

$V = \frac{4}{3}\pi r^3$

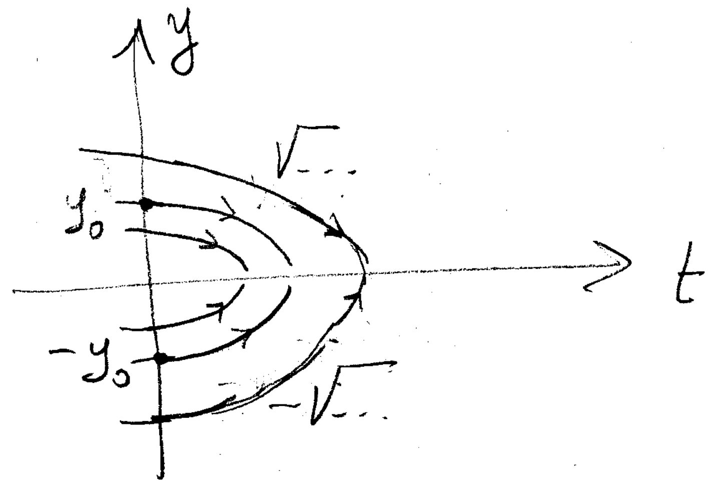
$$\boxed{\frac{dV}{dt} = \pm b A};$$

$A = c V^{2/3}$

4) Multivalued: $\boxed{y' = -\frac{1}{y}} \Longleftrightarrow \frac{dt}{dy} = -y$

singular? regular

GS $\boxed{y^2 = y_0^2 - t}$ $\Longrightarrow$ $\boxed{y = \pm\sqrt{y_0^2 - t}}$



$y = 0$ - singular