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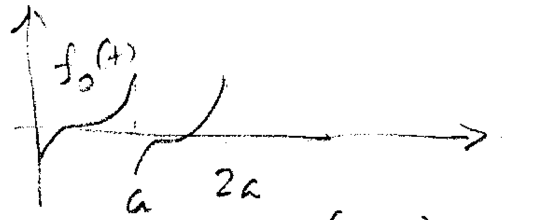
Square wave (6.2: 16, 17 & 19)(1

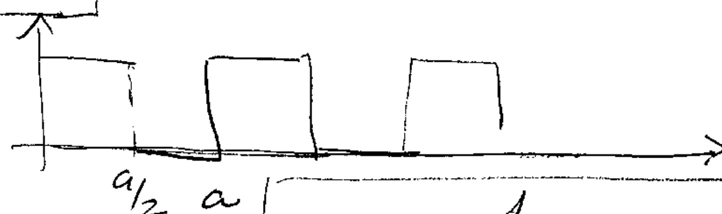
① Periodic: $f(t) = f_0(t-na) = f_0\left(t - a\,\mathrm{floor}\left(\frac{t}{a}\right)\right)$



$$F(s)=\int_0^\infty f(t)e^{-st}dt = \sum_{n=0}^{\infty}\int_{na}^{(n+1)a} f_0(t-na)\,e^{-st}dt = \sum_0^\infty e^{-na}\int_0^a f_0(t)e^{-st}$$

$$\boxed{F(s) = \frac{F_0(s)}{1-e^{-as}}}$$

② Square wave:

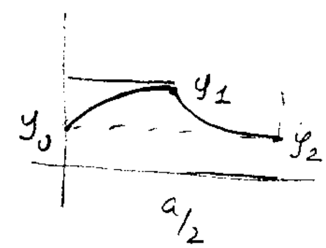


$$F_0(s) = \frac{1-e^{-\frac{as}{2}}}{s} \Rightarrow \boxed{F(s) = \frac{1}{s\left(1+e^{-as/2}\right)}}$$

Periodic solution:

Ask for initial value y0 = y(0), so that 'terminal value (after one period): y(a)=y(0)

$$\begin{cases} y' + ky = SW(t) \\ y(0) = y_0 \; - ? \end{cases}$$



$$y(t) = \begin{cases} \left(y_0 - \frac{1}{k}\right)e^{-kt} + \frac{1}{k}\,; & 0<t<a/2 \\ y_1\, e^{-k(t-a/2)}\,; & \frac{a}{2}<t<a \end{cases} \Rightarrow y_1 = \frac{1}{k} + \left(y_0 - \frac{1}{k}\right)e^{-\frac{ka}{2}}$$

$$\Rightarrow y_0 = \left[\frac{1}{k} + \left(y_0 - \frac{1}{k}\right)e^{-ka/2}\right]e^{-ka/2} = y_2$$

$$\Rightarrow \boxed{y_0 = \frac{e^{-ka/2}}{1+e^{-ka/2}}}$$

Ex: $\begin{cases} y' + y = SW\big|_{a=1} \Rightarrow \\ y_0 = \dfrac{e^{-1/2}}{1+e^{-1/2}} = .3775 \end{cases}$

Same problem for Cos-sourse

$$\boxed{y' + ky = \cos\omega t}$$

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1) Undetermined: $y = \mathrm{Re}\left(\frac{e^{i\omega t}}{k+i\omega}\right) = \boxed{\frac{k\cos\omega t + \omega\sin\omega t}{k^2+\omega^2}} + ce^{-kt}$

y_p periodic

2) Laplace:

$$Y = \frac{1}{(s+k)}\frac{s}{s^2+\omega^2} = \left(\frac{ks+\omega^2}{s^2+\omega^2} - \frac{k}{s+k}\right)\frac{1}{\omega^2+k^2}$$

$$y = \frac{k\cos + \omega\sin}{k^2+\omega^2} - \frac{k}{\omega^2+k^2}e^{-kt}$$

$$\boxed{y(0) = \frac{k}{k^2+\omega^2}}$$ – Unique "periodic" IV