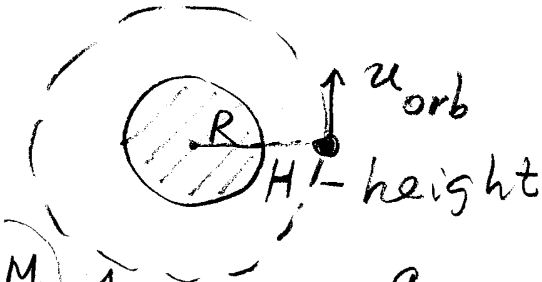
M224   Rocket Science   D. GURARIE

1. Orbital velocity:



$$\underbrace{\frac{u^2}{R+H}}_{\text{centrig. accel.}} = \underbrace{\frac{GM}{(R+H)^2}}_{\text{grav. accel.}} = \frac{GM}{R^2}\frac{1}{(1+H/R)^2} = \frac{g}{(1+H/R)^2}$$

G – grav. const (Newton); M – mass;
g – surface accel.

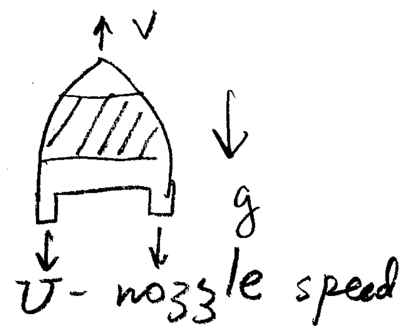
$$\Rightarrow \boxed{u = \sqrt{\frac{gR}{1+H/R}} \approx \sqrt{gR}\left(1 - \frac{1}{2}\frac{H}{R} + \dots\right)}$$

Earth radius $R \approx 6{,}500$ km ; $g \approx 9{,}8$ m/s

$\Rightarrow$ $u_{orb} \approx 2{,}523$ m/s (near surface)

Plot u_{orb} as function of H
compute u_{orb} at $H = 2R$; $3R$

2. Propulsion:



(i) α – fuel Burning rate as fraction of the initial mass m_0

(ii) $m(t) = m_0(1-\alpha t)$ – mass at time t

(iii) Newton law for variable mass (momentum conservation)

$$\boxed{\underbrace{\frac{d}{dt}(mv)}_{\text{rate of change}} = \underbrace{\alpha m_0 U}_{\substack{\text{momentum gained due}\\ \text{to expelled mass}\\ \text{thrust}}} - \underbrace{m g}_{\substack{\text{momentum loss due}\\ \text{to gravity}\\ \text{fall}}}} \qquad (N)$$

(N) is a linear DE:

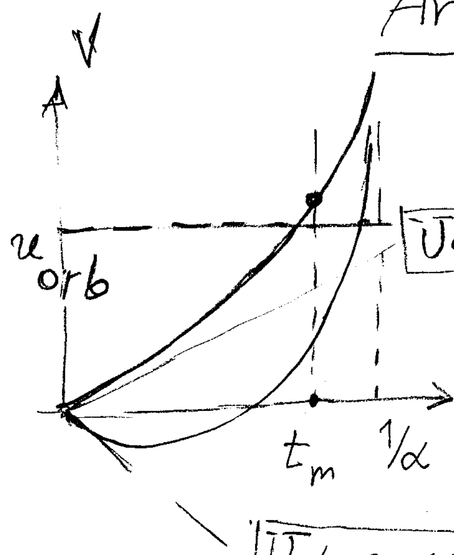
$$\dot{v} + \frac{\dot{m}}{m} v = \frac{\alpha m_0 U}{m} - g$$

$$\boxed{\dot{v} - \frac{\alpha}{1-\alpha t} v = \frac{\alpha U}{1-\alpha t} - g} \rightarrow \text{multiplier } \mu = m(t)\,!$$

Solve (N) by direct integration

$$\boxed{v(t) = \frac{1}{m(t)} \int_0^t (\alpha m_0 U - mg) = \frac{(U\alpha - g)t + g\alpha t^2/2}{1-\alpha t}}$$

Analysis of solution $V(t)$



(i) Solutions blow up at $\boxed{t = 1/\alpha}$

(ii) Take off requires minimal velocity $\boxed{U\alpha - g \geq 0} \Rightarrow \boxed{U_{t/o} \geq g/\alpha}$

(iii) Propulsion time:

$$t_m = \frac{\lambda}{\alpha} = \frac{\text{"fuel fraction of } m_0\text{"}}{\alpha}$$

(iv) To launch sattelite need $\boxed{V(t_m) \geq u_{orb}}$

<u>Example:</u> if $\lambda = .9$ (90% fuel), $\alpha = .05$, then

take off velocity: $U_{t/o} = \frac{g}{\alpha} \approx 200$ m/s

sattelite launch requires:

$$\frac{U\alpha - g}{1-\lambda}\,\frac{\lambda}{\alpha} + \frac{g\alpha}{2}\left(\frac{\lambda}{\alpha}\right)^2 \Big/ (1-\lambda) \geq u_{orb} = \sqrt{gR}$$

$$\Rightarrow \quad U \geq \frac{u_{orb}}{9} + .55\,\frac{g}{\alpha} = \cdots$$

<u>Problem 1:</u> Show that minimal launch nozzle velocity $\boxed{U_{min} \geq \frac{1-\lambda}{\lambda}\, u_{orb} + (1-\lambda/2)g/\alpha}$ for any λ and α.

<u>Problem 2:</u> (i) Find U_{min} to carry 15% load (85% fuel) if $\alpha = .1$ 1/s

Problem 3

(ii) Find time t_m to reach the orbit and its height H (integrate $V(t)$!).

Problem 4:

(i) Derive the DE model for propulsion with friction proportional to v, friction $= \beta v$

(ii) Write its multiplier solution

(iii) Compute it for $\alpha = .1$, friction coeff. $\beta = .01$ and thrust $U = 300$. Determine whether it could reach the orbital speed.