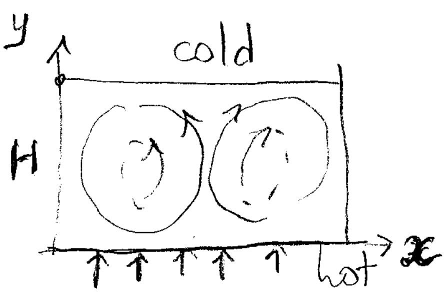
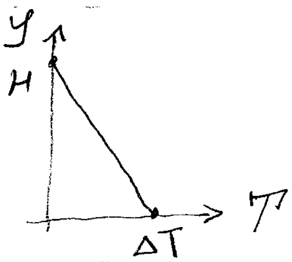
Lorenz-Saltzman (truncated) system for convective motion of thermo-conductive fluid.

D. Gurarie

1) 



temperature profile

$T(y) = \Delta T(1 - y/H)$

* fluid motion: $\boxed{\psi(x,y,t)}$ - stream f-n (circulation)

$\vec{u} = (-\psi_y, \psi_x)$ - velocity field

* temperature: $\Delta T(1 - y/H) + \boxed{\theta(x,y,t)}$

2) Eq-ns of motion (Navier-Stokes)

$$\text{NS} \quad \begin{cases} \partial_t \psi + \dots = F_{viscous} + F_{boyancy}(\theta) \\ \partial_t \theta = \text{transport}(\psi) + \text{thermo-cond.}(\theta) \end{cases}$$

- coupled system of pde's

3) Fourier expansion:

$$\psi(x,y) = (a_{11}\cos x \sin y + \overbrace{\boxed{b_{11}}}^{X}\sin x \sin y) + \dots$$

$$\theta(x,y) = A_{01}\sin y + (\underbrace{\boxed{A_{11}}}_{Y}\cos x \sin y + B_{11}\sin x \sin y) + \underbrace{\boxed{A_{02}}}_{Z}\sin 2y + \dots$$

Lorenz: $\psi = X(t)\sin x \sin y$

$\theta = Y(t)\cos x \sin y + Z(t)\sin 2y$

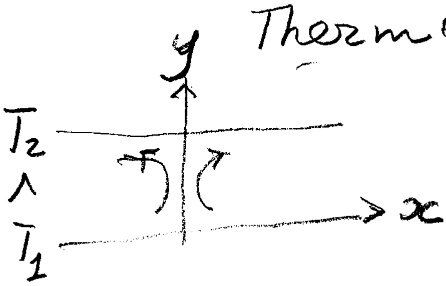
$$\Rightarrow \text{NS} \quad \begin{cases} \dot{X} = \sigma(Y - X) \\ \dot{Y} = \rho X - Y - XZ \\ \dot{Z} = XY - \beta Z \end{cases}$$

DS for 3 amplitude variables: circulation strength X; 2 temperature modes Y, Z

Parameters: $\sigma = \frac{\text{viscosity}}{\text{thermo-cond.}} = 10$; $\beta = \frac{8}{3}$ (geometric);
$\rho = ?$ (bifurcation param. $\propto$ temperat. drop)

Equilibria & stability analysis.

Thermo-convective instability



circulation pattern: $\psi_1(x,y) = \sin\frac{x}{\sqrt{2}} \sin y$

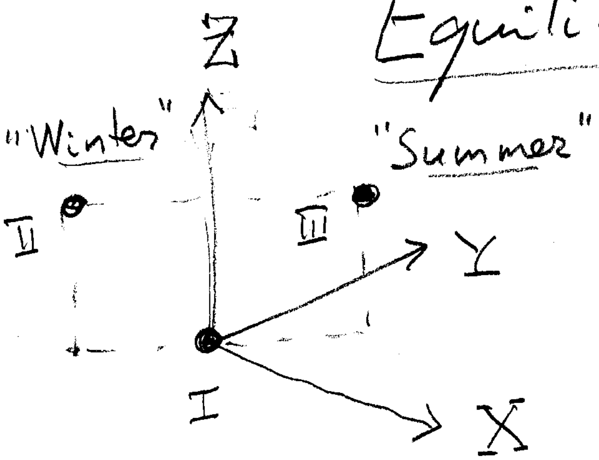
Temperature patterns: $\begin{cases} \theta_1 = \cos\frac{x}{\sqrt{2}} \sin y \\ \theta_2 = \sin 2y \end{cases}$

Time-dependent fields:

stream: $\psi(x,y,t) = X(t)\,\psi_1(x,y)$

Temperature: $\theta(x,y,t) = Y(t)\,\theta_1(x,y) + Z(t)\,\theta_2(x,y)$

Equilibria of 3D Lorenz

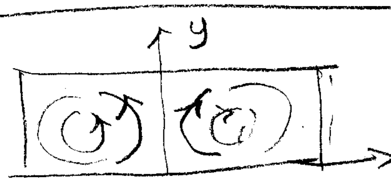


$$\begin{cases} Y = X \\ X[(\rho-1) - Z] = 0 \\ X^2 = \beta Z \end{cases}$$

	X	Y	Z
I	0	0	0
II	$\sqrt{\beta(\rho-1)}$	$\sqrt{\beta(\rho-1)}$	$\rho-1$
III	$-\sqrt{\ldots}$	$-\sqrt{\ldots}$	$\rho-1$

(I) rest

(II) Summer



(III) Winter