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Epidemics: SIR - model

D. GURARIE

$$\boxed{S} \xrightarrow{\beta} \boxed{I} \xrightarrow{\delta} \boxed{R}$$

susceptible infected recovered (immune)

Total pop. density: $S + I + R = N$ - const

Transmission / recovery DS

$$(1)\begin{cases} \dot{S} = -\beta S I + \delta R \\ \dot{I} = \beta S I - \mu I \\ \dot{R} = \mu I - \delta R \end{cases}$$

$\beta = \frac{\text{transm. rate}}{\text{per } I \text{ per } S}$

μ = recovery rate ($\frac{1}{\mu}$ = duration)

δ = loss of immunity rate

Tot. pop. density N - const reduces (1) to 2D system: $R = N - S - I$

$$(2)\begin{cases} \dot{S} = -\beta S I + \delta (N - S - I) \\ \dot{I} = \beta S I - \mu I \end{cases}$$

Plots below illustrate SIR for typical flu parameters: $\mu = .15$ ($\frac{1}{7 \text{ days}}$)

$\delta = .004$ (loss of immunity $\approx$ 250 days)

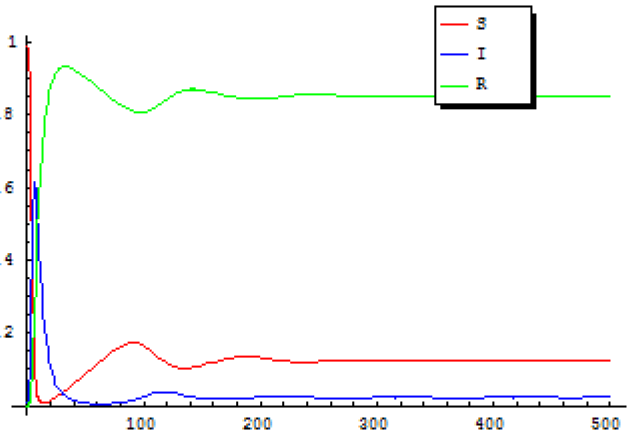
pop. density $N = 1$. Vary β

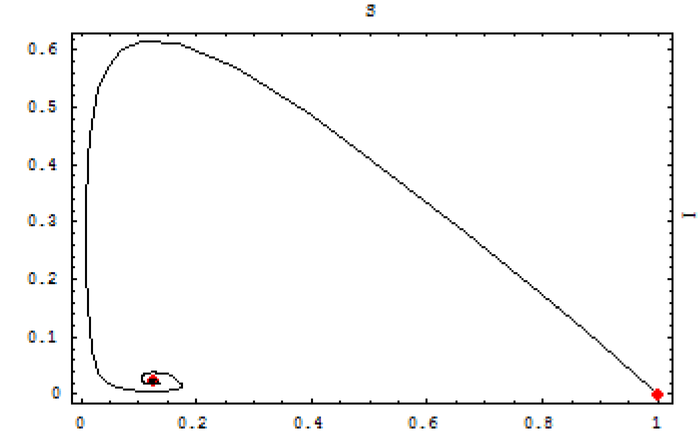
Problem: 1) Find equilibria, plot solution curves and phase-plane for $\beta = 3; 1; .1$

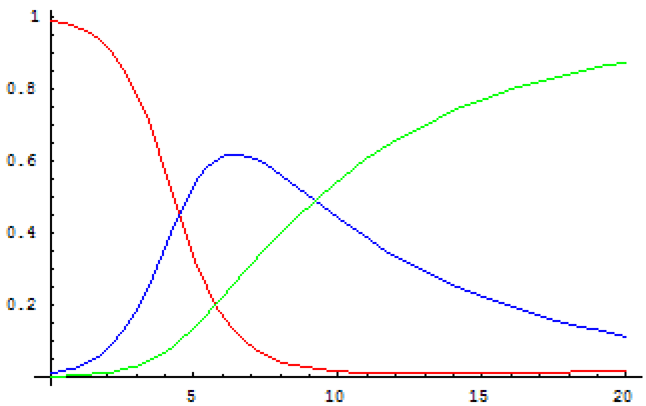
2) Estimate recurrence period (based on plots) in all 3 - cases.

3) Extra: study the effect of delta (= 250,90,30 days) on equilibria and reccurence.

Case 1: β=1.5 (high transmission)







Case 2: β=.3 (low transmission)