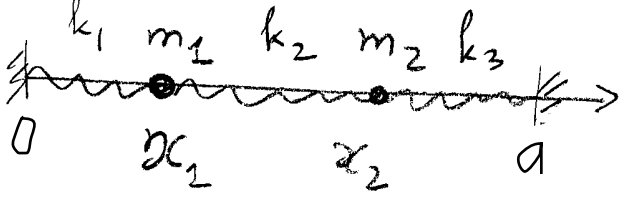
Coupled oscillators



$$\begin{cases} m_1 \ddot{x}_1 = -(k_1+k_2)x_1 + k_2 x_2 \\ m_2 \ddot{x}_2 = k_2 x_1 - (k_2+k_3)x_2 + k_3 a \end{cases}$$

$$\ddot{\underline{Y}} = -\underbrace{\begin{bmatrix} \frac{k_1+k_2}{m_1} & -\frac{k_2}{m_2} \\ -\frac{k_2}{m_2} & \frac{k_2+k_3}{m_2} \end{bmatrix}}_{\underline{A}} \underline{Y} + \begin{pmatrix} 0 \\ F \end{pmatrix} - 2^{nd} \text{ order ODS}$$

Eigenvalue problem for $\underline{A}$:

λ_1	λ_2
X_1	X_2

Always: $0 < \lambda_1 < \lambda_2$; $X_1 = \begin{pmatrix} + \\ + \end{pmatrix}$; $X_2 = \begin{pmatrix} + \\ - \end{pmatrix}$

Example: $\begin{cases} k_1 = k_2 = k_3 = k \\ m_1 = m_2 = m \end{cases} \Rightarrow \underline{A} = \frac{k}{m}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Eigens:

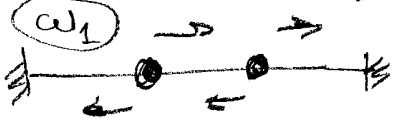
1	3
$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

GS: $\underline{Y}(t) = (c_1 \cos\sqrt{\lambda_1}t + c_2 \sin\sqrt{\lambda_1}t)\underline{X}_1 + (c_3 \cos\sqrt{\lambda_2}t + c_4 \sin\sqrt{\lambda_2}t)\underline{X}_2$

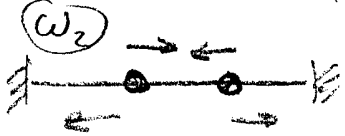
$A_1 \cos(\omega_1 t - \phi_1)$

Frequencies: $\omega_i = \sqrt{\lambda_i}$

Example: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = A_1 \cos(\omega_1 t - \phi_1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} + A_2 \cos(\omega_2 t - \phi_2)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$



X_1 - pattern



X_2 - pattern

Problems:

1) Find an equilibrium position (x_1^0, x_2^0) for coupled oscillator
   (i) in general case $k_1, k_2, k_3, m_1, m_2, a$
   (ii) for $k_1 = k_3 = 1$; $k_2 = 2$; $m_1 = 2$; $m_2 = 1$; $a = 5$

2) Compute general osaillating solution in case 1(ii), and IVP solution for $\begin{cases} x_1(0) = 0 \\ x_2(0) = 4 \end{cases}$ $\begin{cases} \dot{x}_1 = 0 \\ \dot{x}_2 = 0 \end{cases}$

   Plot it.