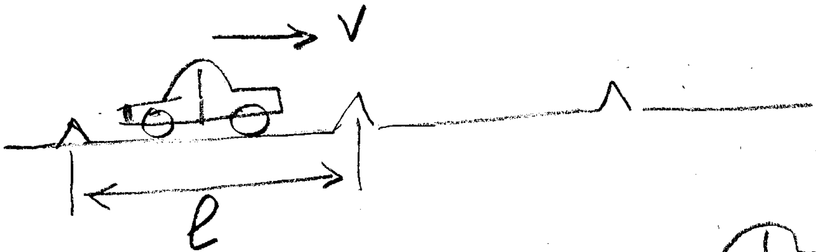
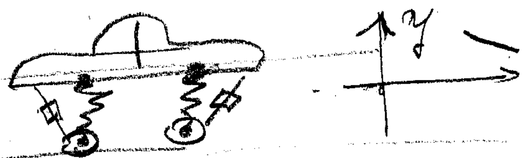
D. GURARIE

Car on a bumpy road



Regularly spaced bumps of length ℓ create periodic delta-source $\boxed{T = \frac{\ell}{v}}$



vertical displacem.

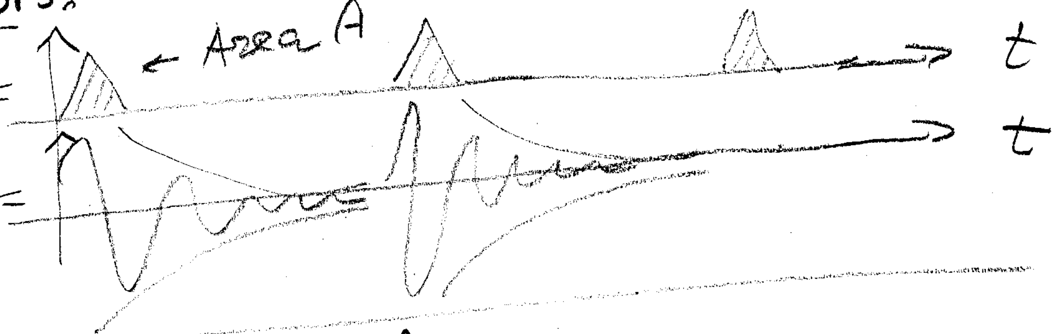
Damped oscillator:

$$\boxed{\ddot{y} + d\dot{y} + ky} = f(t) = A\left[\delta(t) + \delta(t-T) + \dots\right]$$

shock absorb. suspension

$f(t) =$

Expect: $y(t) =$



Laplace method:

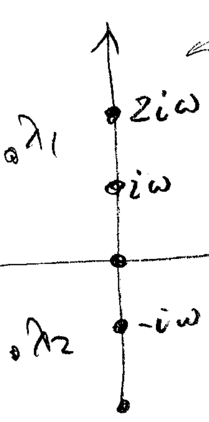
$$F(s) = \frac{A}{1 - e^{-sT}} \Rightarrow Y(s) = \frac{1}{p(s)(1 - e^{-sT})}$$

charact $p(\lambda)$

Need Partial Fraction expansion:

$$Y(s) = \sum \dots \overset{?}{} + \sum \dots$$

complex roots of $(1-e^{-sT})$] complex roots $p(s)$



$$\boxed{\omega = \frac{2\pi}{T}}$$

$$Y(s) = \frac{1}{T}\sum_{m=-\infty}^{\infty} \frac{1}{p(i\omega m)} \frac{1}{(s - i\omega m)} + \left(\frac{A_1}{s-\lambda_1} + \frac{A_2}{s-\lambda_2}\right)$$

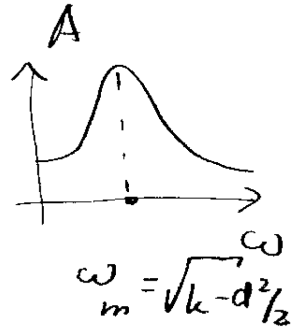
$$y(t) = \frac{A}{T}\left\{\frac{1}{p(0)} + 2\left[\mathrm{Re}\,\frac{e^{i\omega t}}{p(i\omega)} + \mathrm{Re}\,\frac{e^{i2\omega t}}{p(i2\omega)} + \dots\right]\right\}$$

Responce solution & max amplitude

$$y(t) = \underbrace{H\frac{T}{k}}_{y_0} + \left\{ \underbrace{A_1 \cos(\omega t - \phi_1)}_{y_1} + \underbrace{A_2 \cos(2\omega t - \phi_2)}_{y_2} + \dots \right\} + \underbrace{C e^{-\frac{d}{2} t}}_{y_h}$$

Lowest harmonic's amplitude:

$$A_1 = \frac{2TH}{\sqrt{(\omega^2 - k)^2 + (d\omega)^2}} \Rightarrow \max_{\omega} A_1 = \frac{2TH}{d\sqrt{k - d^2/4}}$$



Problem: Find speed $V[m/s]$ that would give max responce, and its amplitude A_1, for a car with $k = 2\ 1/s^2$; $d = .5\ 1/s$ assuming bump spacing $l = 5\ m$ and bump cross section