3D Example

$$A = \left[\begin{array}{cc|c} 2 & -1 & 1 \\ 1 & 2 & 3 \\ \hline 0 & 0 & -2 \end{array}\right] \quad \text{- Block triangular}$$

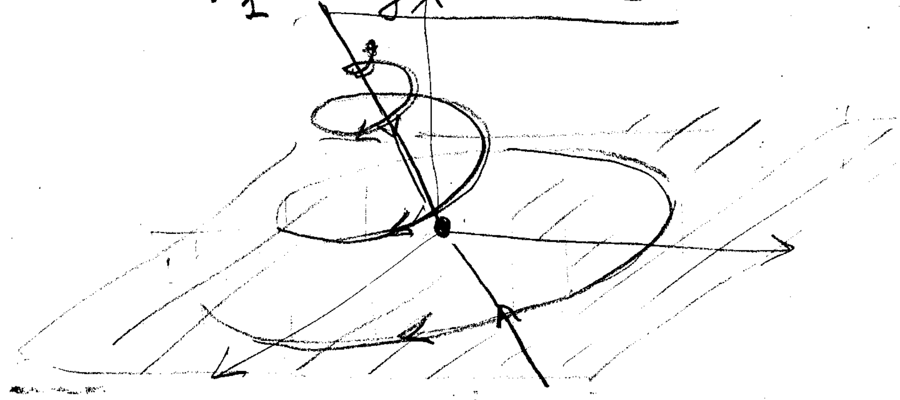
Charact: $p(\lambda) = (\lambda+2)(\lambda^2 - 4\lambda + 5) \Rightarrow \begin{cases} \lambda_1 = -2 \\ \lambda_{2,3} = 2 \pm i \end{cases}$

Eigenvectors: $\boxed{(A - \lambda I)\cdot \underline{X} = \vec{0}}$

$\lambda_1 = -2$	$\lambda_2 = 2 + i$
$(A + 2I)\cdot X_1 = \begin{bmatrix} 4 & -1 & 1 \\ 1 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$(A + \lambda_2 I) = \begin{bmatrix} -i & -1 & 1 \\ 1 & -i & 3 \\ 0 & 0 & 0 \end{bmatrix}$
Geometric: $X_1 = (4, -1, 1) \times (1, 4, 3)$	$X_2 = (-i, -1, 1) \times (1, -i, 3)$
$\boxed{X_1 = (-19, -8, 17)}$	$= \boxed{(-3 + i,\ 1 + 3i,\ 0)}$
	$= U + iV = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$

GS: $\underline{X}(t) = c_1 e^{-2t} X_1 + e^{2t}\left[c_2(U\cos t - V\sin t) + c_3(U\sin t + V\cos t)\right]$

λ_1 - eigen line



2D (complex) plane

Linearized Lorenz

Equilibria

$$\text{V.F.}\quad F(\underline{X}) = \begin{pmatrix} \sigma(y-x) \\ \rho x - y - xz \\ -\beta z + xy \end{pmatrix} \Rightarrow \begin{cases} (0,0,0) \\ (\pm\sqrt{\beta(\rho-1)};\ \pm\sqrt{\dots};\ \rho-1) \end{cases}$$

if $\rho > 1$

For linearized DS use

Jacobian matrix: $A = \left(\dfrac{\partial f_i}{\partial x_j} \right)$; $F = \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}$ $X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

Lorenz: $A = \begin{bmatrix} -\sigma & \sigma & 0 \\ (\rho - z) & -1 & -x \\ y & x & -\beta \end{bmatrix}$

equil.	$(0,0,0)$	$(\sqrt{\dots}, \sqrt{\dots}, \rho-1)$
A	$\begin{bmatrix} -\sigma & \sigma & 0 \\ \rho & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix}$	$\begin{bmatrix} -\sigma & \sigma & 0 \\ 1 & -1 & \sqrt{\dots} \\ \sqrt{\dots} & \sqrt{\dots} & -\beta \end{bmatrix}$
EV:		