Summary of Hamiltonian & gradient DS

Gura

Def.	$\mathbb{F} = \begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} -h_y \\ h_x \end{pmatrix} = \nabla h^{\perp}$ $h(x,y)$ – Ham. f-n	$\mathbb{F} = \begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} S_x \\ S_y \end{pmatrix} = \nabla S$ $S(x,y)$ – pot. f-n
Test	$\boxed{\nabla \circ \mathbb{F} = f_x + g_y = 0}$	$\boxed{-f_y + g_x = 0}$
Antidiff.	$h(x,y) = \int_C -f\,dy + g\,dx$ – line	$S(x,y) = \int_C f\,dx + g\,dy$
2-step	1° $h_y = -f \Rightarrow \boxed{h = -\int f\,dy + \overset{?}{C(x)}}$ 2° $h_x = \dots + C'(x) = g \Rightarrow C(x) = \dots$	$S_x = f \Rightarrow \boxed{S = \int f\,dx + \overset{?}{C(y)}}$ $S_y = \dots + C'(y) = g \Rightarrow C(y) = \dots$
Examples	1) $\mathbb{F} = (f(y), g(x)) \Rightarrow h = \dots$	$\mathbb{F} = (f(x), g(y)) \Rightarrow S = \dots$
Linear	2) $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ – traceless $\nabla \circ \mathbb{F} = \operatorname{tr} A = 0$ $\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}$	$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ – symmetric $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$
Equilibria	= crit p-ts (h)	= crit p-ts (S)
Jacobian	$J = \begin{bmatrix} -h_{xy} & -h_{yy} \\ h_{xx} & h_{xy} \end{bmatrix}$ $\det J = h_{xx} h_{yy} - h_{xy}^2$	$J = \begin{bmatrix} S_{xx} & S_{xy} \\ S_{xy} & S_{yy} \end{bmatrix}$ – Hessian $\det = S_{xx} S_{yy} - S_{xy}^2$
Types	$\begin{rcases} \text{min}(h) \\ \text{max}(h) \end{rcases}$ $\det J > 0 \Rightarrow$ cent. saddle (h) $\Longleftrightarrow$ saddle	min S – source max S – sink saddle (S) = saddle ($\mathbb{F}$)