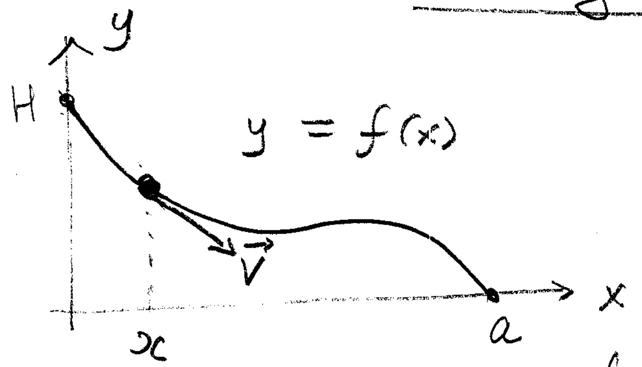
D. GURARIE

Sliding down the slope



Time $= \dfrac{\text{distance}}{\text{speed}}$; $\boxed{dt = \dfrac{ds}{V}}$

1) For ideal slope (no friction) speed V depends on height y only, due to energy conservation (kinetic + potent.)

$$\boxed{E = \frac{mV^2}{2} + mg\,y}$$ — const; $E = mgH$ — pot. energy at the top.

$\Rightarrow \quad V = \sqrt{2g(H-y)}$

2) Parametrize motion by horizontal coord.

$(x(t), f(x(t))) \longrightarrow \vec{V} = \dot{x}\,(1, f'(x))$ — vel.

Speed $\quad V = \dot{x}\sqrt{1+f'^2}$.

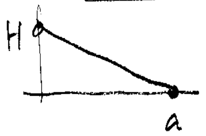
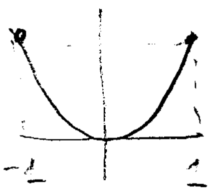
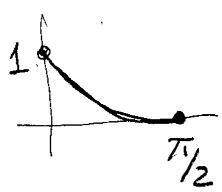
Get autonomous DE for $x(t)$

$$\frac{dx}{dt} = \sqrt{2g\left(\frac{H - f(x)}{1+f'(x)^2}\right)} = F(x)$$

Solve by separation:

time to slide to x $\quad \boxed{\displaystyle\int_0^x \frac{dx}{F(x)} = t}$ $\qquad$ Total time $\quad \boxed{\displaystyle\int_0^a \frac{dx}{F(x)} = T}$

Examples:

$f(x)$	$F(x)$	integral	solution t
$H-kx$ 	$\sqrt{2g\frac{kx}{1+k^2}}$	$\int\frac{dx}{\sqrt{x}}=\sqrt{\frac{2gk}{1+k^2}}\,t$	$2\sqrt{x}=\sqrt{\frac{2gk}{1+k^2}}\,t+c \Rightarrow$ $x=\left(c+\sqrt{\frac{g}{2}\frac{k}{1+k^2}}\,t\right)^2$
$x^2/2$ 	$\sqrt{2g\,\frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)}$	$\int\sqrt{\frac{1+x^2}{1-x^2}}\,dx=\sqrt{g}\,t$	$E(x,-1)=\sqrt{g}\,t$ ↑ elliptic integral
$1-\sin x$ 	$\sqrt{2g\,\frac{\sin x}{1+\cos^2 x}}$	$\int\sqrt{\frac{1+\cos^2 x}{\sin x}}\,dx=\sqrt{2g}\,t$	$? \quad =\sqrt{2g}\,t$