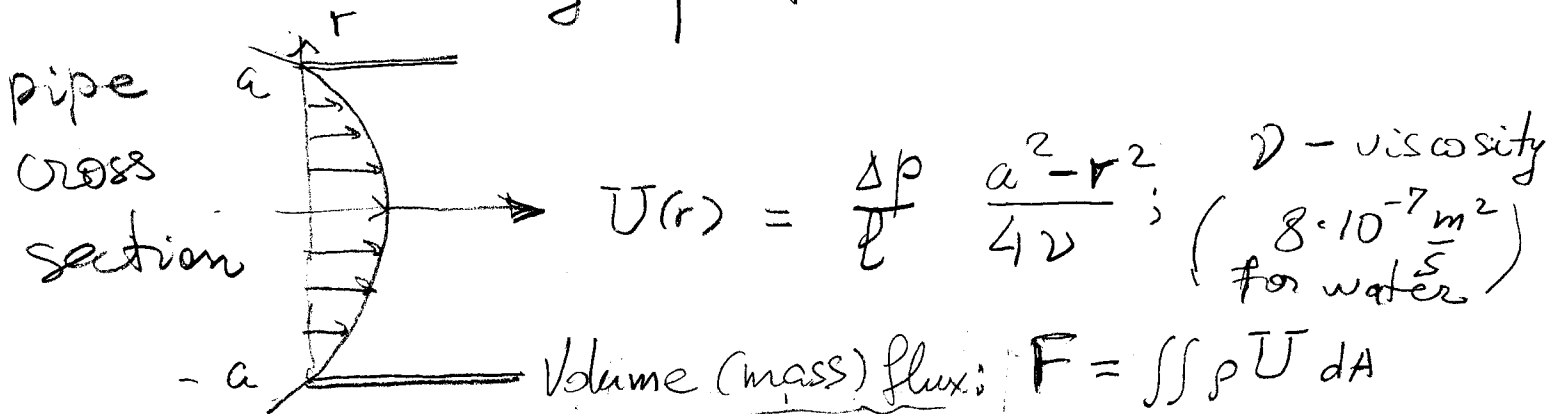


Poiseuille pipe-flow & CR circuits

① Poiseuille — a stationary fluid flow

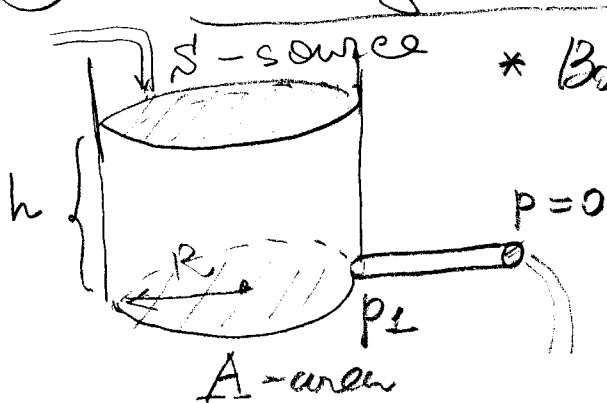
in pipe of radius a
 (cross section πa^2) created
 by pressure gradient $\frac{\Delta p}{l} = \frac{p_1 - p_2}{l}$
 has velocity profile:



$$F = \frac{\pi a^4}{8} \frac{\Delta p}{l \nu} \rho$$

ρ - mass density

② Leaking tank:



* Bottom (hydrostatic) pressure:

$$p_1 = \rho g h; \quad p_2 = 0$$

Volume loss: $\frac{dV}{dt} = F$

$$A \frac{dh}{dt} = - \frac{\pi a^4 g}{8 l \nu} h + S$$

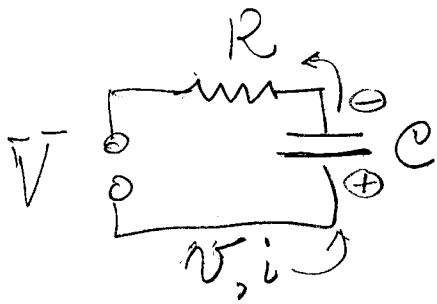
Linear 1st order DE: $h' = -a h + b$

Problem: (i) How long does it take to drain ⁽²⁾ half tank of water (kinematic viscosity $\nu = 8 \cdot 10^{-7} \text{ m}^2/\text{s}$) filled at height $h_0 = 5 \text{ m}$ through a pipe of radius $a = 2 \text{ cm}$, length $l = 2 \text{ m}$

(ii) Can you empty the tank?

(iii) The water is supplied to the tank at a rate $S = 0.3 \text{ m}^3/\text{s}$ and drained through the same pipe. What equilibrium level h it will attain?

CR circuits (Kirchoff) (3)



i - current (flow of charge)
 v - voltage (potential force driving charge)
 q - charge (accumulating at C)

Kirchoff laws:

(i) $V = Ri$ (ohm) - voltage applied to R creates current proportional to V/R
 R - resistance.

(ii) $q = Cv$ - charge accumulation creates voltage proportional to q/C
 C - capacitance.

(iii) $\frac{dq}{dt} = -i$ - current is due to "moving charges" on C

$$C \frac{dv}{dt} = -\frac{v}{R} \Rightarrow v' = -\frac{1}{RC} v + \frac{V}{RC}$$

voltage source

Hydrodynamic analogy

- * voltage = pressure drop (\propto height h in tank)
- * current = velocity U (or volume flux F)
- * charge = mass/volume

(i) current $\rightarrow F = \frac{\rho a^4 g}{8\eta l} h \leftarrow \text{voltage} \cdot \frac{1}{R}$

(ii) mass/vol. = $A h$
 capacitance \rightarrow

Problem 2: Set up a DE model for (4) 2-tank system connected by pipes of radii (a_1, a_2) , length l_1, l_2 , and source S

