Prove the Lyapounov central limit theorem:

For each $n \geq 1$, let $\{X_{n,i}\}_{i=1}^{k_n}$ be a family of independent random variables. Suppose that $\mathbb{E}X_{n,i} = 0$ for all $n, i$ and that there is a $\delta > 0$ such that for each $n, i$, $\mathbb{E}|X_{n,i}|^{2+\delta} < \infty$. Let $S_n := \sum_{k=1}^{k_n} X_{n,k}$ and let $s_n^2 := \sum_{k=1}^{k_n} \mathbb{E}X_{n,k}^2$. If

$$\frac{1}{s_n^{2+\delta}} \sum_{k=1}^{k_n} \mathbb{E}|X_{n,k}|^{2+\delta} \xrightarrow{n\to\infty} 0,$$

then $\frac{S_n}{s_n}$ converges weakly to a standard Gaussian random variable.