Math 423 Homework 8

1. Let $f(x) = \prod_{j=1}^{n} x_j^{\alpha_j}$ for $\alpha_j \in \mathbb{N} \cup \{0\}$ (so f is a monomial on $\mathbb{R}^n$). Show that $\int f d\sigma = 0$ if any of the α_j is odd, and if all the α_j are even, then

$$\int f d\sigma = \frac{2\Gamma(\beta_1)\cdots\Gamma(\beta_n)}{\Gamma(\beta_1 + \cdots + \beta_n)},$$

   where $\beta_j = \frac{\alpha_j+1}{2}$.

   Hint: use the same idea that we used to find the surface area of the sphere: calculate $\int f(x)e^{-x^2}dx$ in two ways.

2. Let ν be a signed measure.

   (a) Show that E is ν-null if and only if $|\nu|(E) = 0$.

   (b) If μ is another signed measure, show that $\nu \perp \mu$ if and only if $|\nu| \perp \mu$, if and only if $\nu^+ \perp \mu$ and $\nu^- \perp \mu$.

3. Let μ be a positive measure on $(X, \mathcal{M})$, and let f be an extended μ-integrable function. Define $\nu(E) = \int_E f d\mu$. Describe the Hahn decompositions of X for ν and the positive, negative, and total variations of ν in terms of f and μ.