

Math 423 Homework 5

1. Let $f \in L^+$ and for $E \in \mathcal{M}$, define $\lambda(E) := \int_E f d\mu$. Show that λ is a measure on \mathcal{M} and that, for any $g \in L^+$, $\int g d\lambda = \int f g d\mu$.
2. Prove the monotone convergence theorem from Fatou's lemma.
3. Suppose that $\{f_n\} \subseteq L^1(X)$ and that $f_n \rightarrow f$ uniformly.
 - (a) Show that if $\mu(X) < \infty$, then $f \in L^1(X)$ and $\int f_n d\mu \rightarrow \int f d\mu$.
 - (b) Show that the conclusions above can fail if $\mu(X) = \infty$.
4. Compute (with justification!) $\lim_{n \rightarrow \infty} \int_0^\infty \left(1 + \frac{x}{n}\right)^{-n} \sin\left(\frac{x}{n}\right) dx$.