

Math 423 Homework 13: additional problems

1. Let $f \in L^+(\mathbb{R}^n)$. Show that the measure $f dm$ is regular if and only if $f \in L^1_{loc}$.
2. Show that if λ and μ are positive, mutually singular Borel measures on \mathbb{R}^n and $\lambda + \mu$ is regular, then so are λ and μ .
3. (a) Let

$$F(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0; \\ 0, & x = 0. \end{cases}$$

For which $a < b$ is $F \in BV([a, b])$? Is $F \in BV$?

- (b) Now answer the same questions for

$$G(x) = \begin{cases} \frac{\sin(x)}{x}, & x \neq 0; \\ 1, & x = 0. \end{cases}$$