# Math 307 Homework
# September 21, 2015

1. Prove that if a list of vectors $(\mathbf{v}_1, \ldots, \mathbf{v}_n)$ spans $\mathbb{F}^m$, then $m \leq n$.

2. Is it true that the set of all eigenvectors of a linear map $\boldsymbol{T} \in \mathcal{L}(V)$, together with the 0 vector, must be a subspace of $V$? Give a proof or a counterexample.

3. Let $\boldsymbol{D}$ be the differentiation operator on $C^\infty(\mathbb{R})$, the space of infinitely differentiable functions on $\mathbb{R}$. What is the kernel of $\boldsymbol{D}$?

4. If $U_1$ and $U_2$ are both subspaces of $V$, then we define

$$U_1 + U_2 := \{u_1 + u_2 | u_1 \in U_1, u_2 \in U_2\}.$$

   (a) Suppose that $\boldsymbol{S}, \boldsymbol{T} \in \mathcal{L}(V, W)$. Prove that

$$\text{range}(\boldsymbol{S} + \boldsymbol{T}) \subseteq (\text{range}\, \boldsymbol{S}) + (\text{range}\, \boldsymbol{T}).$$

   (b) Suppose that $\mathbf{A}, \mathbf{B} \in \mathrm{M}_{m,n}(\mathbb{F})$. Prove that

$$C(\mathbf{A} + \mathbf{B}) \subseteq C(\mathbf{A}) + C(\mathbf{B}).$$