Math 307 Homework
September 16, 2015

1. Consider the linear map $\boldsymbol{T} : \mathbb{R}^2 \to \mathbb{R}^2$ whose matrix is

$$\begin{bmatrix} -1 & \frac{3}{2} \\ 0 & 2 \end{bmatrix}.$$

   (a) Show that $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ are eigenvectors, and determine the corresponding eigenvalues.

   (b) Draw the image of the unit square $\{(x, y)|0 \le x, y \le 1\}$ under $\boldsymbol{T}$.

2. Let $\boldsymbol{T} : \mathbb{R}^2 \to \mathbb{R}^2$ be the map defined by first rotating counterclockwise by θ and then reflecting across the line $y = x$. Find the matrix of $\boldsymbol{T}$.

3. Define $\boldsymbol{T} : C[0, \infty) \to C[0, \infty)$ by

$$\boldsymbol{T}f(x) = \int_0^x f(y)\ dy.$$

   (Note that by the Fundamental Theorem of Calculus, $\boldsymbol{T}f$ is an antiderivative of f with $\boldsymbol{T}f(0) = 0$.)

   (a) Show that $\boldsymbol{T}$ is linear.

   (b) Show that $\boldsymbol{T}$ is an integral operator (as discussed in class), although with a discontinuous kernel k.