Math 307 Homework
November 9, 2015

1. Let V be a finite dimensional complex inner product space. Prove that if $\boldsymbol{T} \in \mathcal{L}(V)$ is normal, then there is an operator $\boldsymbol{S} \in \mathcal{L}(V)$ such that $\boldsymbol{T} = \boldsymbol{S}^2$.

   Hint: Every complex number has a complex square root.

2. Prove that if $\mathbf{A} \in \mathrm{M}_n(\mathbb{F})$ is Hermitian, then there are an orthonormal basis $(\mathbf{v}_1, \ldots, \mathbf{v}_n)$ of $\mathbb{F}^n$ and numbers $\lambda_1, \ldots, \lambda_n \in \mathbb{R}$ such that

$$\mathbf{A} = \sum_{j=1}^{n} \lambda_j \mathbf{v}_j \mathbf{v}_j^*.$$

3. Find the singular value decomposition of $\begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}$.

   Hint: The necessary tools are in Proposition 3.37 and its proof.