Math 307 Homework
November 6, 2015

1. Prove that if $\mathbf{A} \in M_n(\mathbb{C})$ has singular values $\sigma_1, \ldots, \sigma_n$, then

$$|\operatorname{tr} \mathbf{A}| \leq \sum_{j=1}^{n} \sigma_j.$$

   Hint: Use SVD (in the form of Corollary 3.31) and the Cauchy–Schwarz inequality.

2. Suppose that V is a complex inner product space, $\boldsymbol{T} \in \mathcal{L}(V)$, and $\boldsymbol{T}^* = -\boldsymbol{T}$. Prove that every eigenvalue of $\boldsymbol{T}$ is purely imaginary (that is, of the form ia for some $a \in \mathbb{R}$).

3. Let V and W be finite dimensional inner product spaces and let $\boldsymbol{T} \in \mathcal{L}(V, W)$. Prove that
$$\ker \boldsymbol{T}^* = (\operatorname{range} \boldsymbol{T})^{\perp}.$$