Math 307 Homework
October 7, 2015

1. Let P be the plane

$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, 3x - 2y + z = 0 \right\}.$$

   (a) Find a basis for P.

   (b) Determine whether each of the following vectors is in P. For each one that is, give its coordinate representation in terms of your basis.

   i. $\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

   ii. $\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$

   iii. $\begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$

2. (a) Show that $\mathcal{B} = \left(1, x, \frac{3}{2}x^2 - \frac{1}{2}\right)$ is a basis of $\mathcal{P}_2(\mathbb{R})$.

   (b) Find the coordinate representation of x^2 with respect to $\mathcal{B}$.

   (c) Let $\boldsymbol{D} : \mathcal{P}_2(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$ be the derivative operator. Find the coordinate representation of $\boldsymbol{D}$ with respect to $\mathcal{B}$ (i.e., with the same basis $\mathcal{B}$ on both the domain and the codomain). Use it to calculate $\frac{d}{dx}[x^2]$.