Math 307 Homework
October 12, 2015

1. Let $\mathcal{B} = \left(1, x, x^2\right)$ and let $\mathcal{B}' = \left(1, x, \frac{3}{2}x^2 - \frac{1}{2}\right)$ in $\mathcal{P}_2(\mathbb{R})$. Find the change of basis matrices $[\boldsymbol{I}]_{\mathcal{B},\mathcal{B}'}$ and $[\boldsymbol{I}]_{\mathcal{B}',\mathcal{B}}$.

2. Let $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

   (a) Find all the eigenvalues and eigenvectors of $\mathbf{A}$.
   Hint: You can just work directly with the definition here: suppose that $\mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$ and see what you can say about x, y, and λ. Or you can think about under what circumstances $\mathbf{A} - \lambda \mathbf{I}_n$ has rank smaller than 2.

   (b) Prove that $\mathbf{A}$ is not diagonalizable (no matter what $\mathbb{F}$ is).

3. Let $\boldsymbol{P} : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by orthogonal projection onto the line $y = -x$.

   (a) Find a basis $\mathcal{B}$ of $\mathbb{R}^2$ such that $[\boldsymbol{P}]_{\mathcal{B}}$ is diagonal.

   (b) Compute the change of basis matrices $[\boldsymbol{I}]_{\mathcal{B},\mathcal{E}}$ and $[\boldsymbol{I}]_{\mathcal{E},\mathcal{B}}$ (make sure you're clear about which is which!).

   (c) Find the matrix $[\boldsymbol{P}]_{\mathcal{E}}$ of $\boldsymbol{P}$ with respect to the standard basis.