Homework 31

1. Show that similar matrices have the same eigenvalues.

2. Let $A \in M_n(\mathbb{C})$ have singular values $s_1, \dots, s_n$, and define $B \in M_{2n}(\mathbb{C})$ by

$$B = \begin{bmatrix} 0 & A \\ A^* & 0 \end{bmatrix}.$$

   Show that the eigenvalues of B are $s_1, \dots, s_n, -s_1, \dots, -s_n$. *Hint:* Use the singular value decomposition of A.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by reflecting across the line making angle θ with the x-axis. Find the matrix of T in a convenient basis, and then use it to find the matrix of T with respect to the standard basis.

4. Let S be an invertible linear map on a finite-dimensional vector space V and $\{e_i\}_{i=1}^n$ a basis of V. Show that $\{Se_i\}_{i=1}^n$ is also a basis of V and that

$$\mathcal{M}(I, \{Se_i\}, \{e_i\}) = \mathcal{M}(S, \{e_i\}).$$