

Name: \_\_\_\_\_

## Math 224 Quiz 3 – E. Meckes

1. Consider the non-linear system

$$\begin{aligned}\frac{dx}{dt} &= y - ax^3 \\ \frac{dy}{dt} &= y - x,\end{aligned}$$

where  $a$  is a parameter.

- (a) Identify the nullclines of the system. Then find all the equilibria of the system, and identify the value(s) of  $a$  at which the number and/or location of the equilibria changes.
- (b) Sketch the nullclines for  $a = -1$ ,  $a = 0$ , and  $a = 1$ . You do not need to draw arrows, just the nullclines themselves. Please label everything in your pictures!

(c) For  $a = 1$ , compute the Jacobian of the system.

(d) Linearize the system at each of the equilibria and identify the type of equilibrium. You do not need to calculate eigenvectors.

- (e) Sketch the phase portrait of the system with  $a = 1$ , making use of your answers to the previous parts. Again, you do not need to calculate eigenvectors.

2. Consider the undamped, forced harmonic oscillator modeled by

$$\frac{d^2y}{dt^2} + 4y = 2 \cos(\omega t).$$

(a) If  $\omega \neq 2$ , find the general solution to the equation above.

(b) If  $\omega = 3$ , give the solution to the initial value problem

$$\frac{d^2y}{dt^2} + 4y = 2 \cos(\omega t), \quad y(0) = y'(0) = 0.$$

(c) Now suppose  $\omega = 2$ . Find the solution to the initial value problem

$$\frac{d^2y}{dt^2} + 4y = 2 \cos(\omega t), \quad y(0) = y'(0) = 0.$$

(d) Describe the long-term behavior of your solution above, and give a rough sketch.