Name: _____

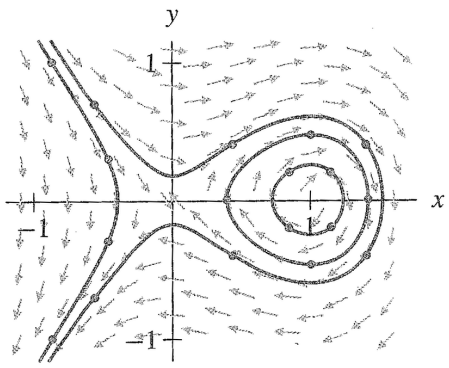
Math 224 Practice Quiz 2

1. Solve the system

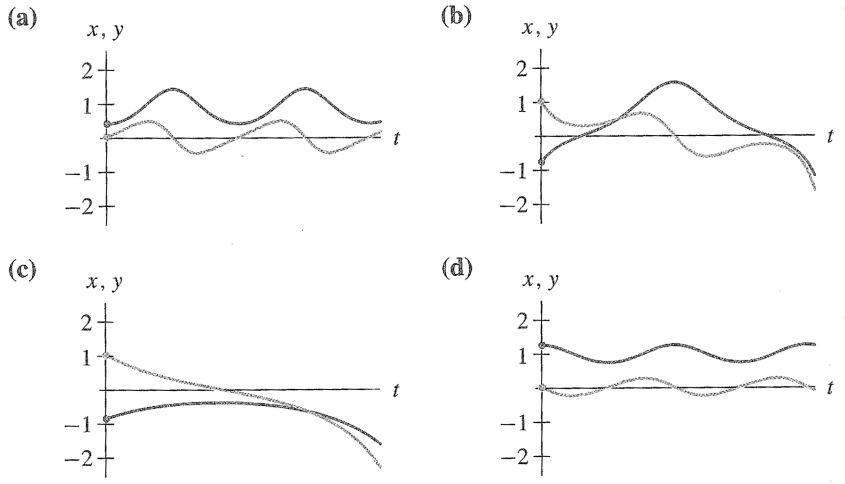
$$\frac{dx}{dt} = 3x + y,$$
$$\frac{dy}{dt} = -y$$

with initial conditions $x(0) = 1$, $y(0) = 2$.

2. The following is a graph of four solution curves $(x(t), y(t))$ to an autonomous system of differential equations, together with the direction field of the system.



Below are four pairs of graphs of $x(t)$ and $y(t)$ versus t.



Match each of the pairs of graphs to a solution curve in the phase plane. Label which graph is x and which is y. Finally, describe the long-term behavior of solutions in all cases.

3. Consider the linear system $\dfrac{d\mathbf{Y}}{dt} = \mathbf{BY} = \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix} \mathbf{Y}$.

(a) Find the eigenvalues of B.

(b) Find the corresponding eigenvectors.

(c) Give the general solution to the system.

(d) Solve the initial value problem $\dfrac{d\mathbf{Y}}{dt} = \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix} \mathbf{Y}$ and $\mathbf{Y}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

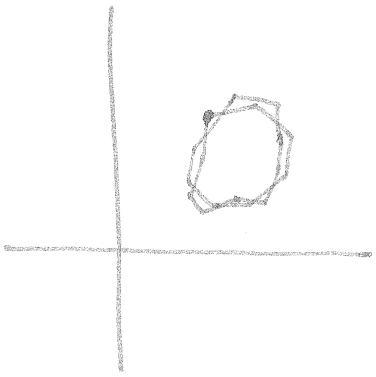
(e) What is the long-term behavior of your solution as $t \to \infty$? What about $t \to -\infty$?

(f) Sketch the phase plane for this system. Make sure to include any straight-line solutions, indicate direction of solution curves in time, and include the solution curve you found above to the initial value problem.

4. Suppose you used Euler's method to approximate the solution to the **autonomous** system

$$\frac{d\mathbf{Y}}{dt} = \mathbf{F}(\mathbf{Y})$$

with initial condition $\mathbf{Y}(0) = \mathbf{Y}_0$, and the resulting solution curve plotted on the phase plane looked like this:



(a) Explain how you can tell that the Euler's method approximation must not be a very good approximation of the true solution.

(b) What would you do to try to get a better approximation?

(c) What do you guess the true solution looks like, based on the approximation above?