Name: _____

Math 224 Exam 2
February 18, 2015

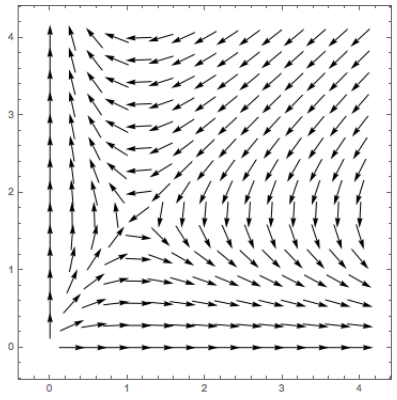
1. Consider following two population models

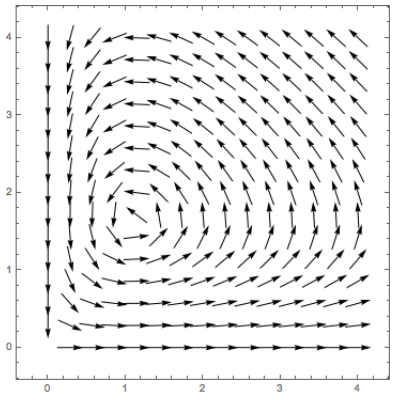
$$\frac{dx}{dt} = 2x - 1.2xy, \qquad \frac{dx}{dt} = 2x - 1.2xy,$$
$$\frac{dy}{dt} = -y + 0.9xy, \qquad \frac{dy}{dt} = y - 0.9xy.$$

   (a) One of the models is a predator/prey system, and the other models two competing species. Which is which (explain your answer)?

   (b) For the predator/prey system, which variable represents the predators, and which represents the prey (explain)?

(c) Here are the direction fields for the two systems. Identify which direction field goes with which system.





(d) For each system, sketch a solution curve on the direction field corresponding to the initial condition $\left(2, \frac{3}{2}\right)$. Describe the long-term behavior of the populations in both systems.

2. Give the general solution to

$$\frac{dx}{dt} = 3x + y,$$
$$\frac{dy}{dt} = -y.$$

3. Solve the initial value problem

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y = 0, \qquad y(0) = 0, \quad y'(0) = 3.$$

4. Usually in zombie movies, zombies do not stop infecting new victims until they are destroyed by a human; humans destroy as many zombies as they can. This leads us to the following variation of the SIR model (where H is the fraction of the initial population made of humans, Z is the fraction made of zombies, and $D = 1 - H - Z$ is the fraction of dead zombies, which we need not include explicitly):

$$\frac{dH}{dt} = -\alpha HZ,$$
$$\frac{dZ}{dt} = \alpha HZ - \gamma H.$$

(a) Calculate the equilibrium points of the model.

(b) Find the region of the phase plane where $\frac{dZ}{dt} > 0$.

(c) Suppose that $\frac{\gamma}{\alpha} < 1$. Sketch the part of the phase portrait of the system where H and Z are positive. What does the model predict will happen to the human/zombie population?